Market and Information Economics

Preliminary Examination

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Instructions: This examination consists of six questions. <u>You must answer the first question</u> and you must answer <u>four of the remaining five</u> questions (i.e. answer four of the questions numbered 2-6). Each question answered (five in total) has a weight of 20% in the final examination score. Please read through the entire examination before making a decision on the particular set of five questions you actually answer. The examination proctor will review the content of the exam at the beginning of the time period (9:00 am). He or she will answer general questions for the entire set of students writing this prelim. You have until 1:15 pm to complete the exam. Good Luck!

You Must Answer this Question

1. Suppose there are only two types of agents who are distinguished by their cost functions and let the action of an agent be the amount of output he produces. Let x_t and $c_t(x)$ be the output and cost function of an agent of type t. For definitness, let agent 2 be the high-cost agent, so that $c_2(x) > c_1(x)$ for all x. Let $s(x)$ be the payment as a function of output. Suppose that agent t's utility function is of the form $s(x) - c_t(x)$, and the utility function of the principal is $x - s(x)$. The principal is unsure of the type of agent he faces, but he attaches a probability of π_t that it is type t. Assume the single-crossing property of the cost function, that is, the agent with higher total costs also has higher marginal costs; i.e., that $c_2'(x) > c_1'(x)$ for all x.

(a) Prove the following implications of the single-crossing property:

(i) If $c_2'(x) > c_1'(x)$ for all x, then for any two distinct levels of output x_1 and x_2, for which $x_2 > x_1$, we must have $c_2(x_2) - c_2(x_1) > c_1(x_2) - c_1(x_1)$.

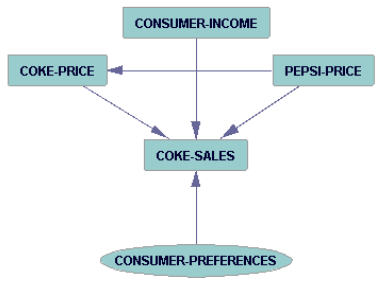
(ii) If $c_2(x) > c_1(x)$ and $c_2'(x) > c_1'(x)$ then any two indifference curves of a type 1 agent and a type 2 agent intersect at most once.

(b) Write down the principal's optimization problem with the participation contraints and the incentive compatibility constraints.

(c) For simplicity, suppose that $c_t(x_t) = \frac{tx_t^2}{2}$ and $\pi_1 = \pi_2 = \frac{1}{2}$. Find the optimal incentive plan that solves this optimizaiton problem. Calculate the principal's profit.

Answer four of the following five questions

2. Consider the following the graph that generates variables on Coke Sales at a supermarket as a function of Coke Price, Pepsi Price, Consumer Income and an Unobserved Latent Variable called Consumer Preferences. [Note: Coke and Pepsi refer to names of soft drinks (non-alcoholic beverages); also known as Coca Cola and Pepsi Cola, respectively.]



i. Specify an equation (or set of equations) to find an unbiased and consistent estimate of how Coke Sales will change due to a change in Pepsi Price (Total not Partial Effect). Indicate the estimated coefficient or set of coefficients we must focus on to obtain the intended estimate.

ii. Specify an equation (or set of equations) to find an unbiased and consistent estimate of how Coke Sales will change due to a change in Coke Price (Total not Partial Effect). Indicate the estimated coefficient or set of coefficients we must focus on to obtain the intended estimate.

iii. Consider the cross price elasticity: $\xi_{C,P} = (\partial Q_{coke}/\partial P_{Pepsi}) \times (P_{Pepsi}/Q_{coke})$, where Q_{coke} refers to the quantity of Coke sold and P_{Pepsi} refers to the Price of Pepsi. How does one obtain the components to estimate this cross price elasticity? That is specify an estimation strategy to obtain the components of the elasticity? Indicate the estimated coefficient or set of coefficients we must focus on to obtain the intended estimate

iv. Suppose that the graph given above is amended such that there is an additional edge (arrow) running from Consumer Preferences to Pepsi-Price (Consumer Preferences → Pepsi Price). Marketers of Pepsi see that there is increased awareness and excitement by consumers about consuming Pepsi. Marketers of Pepsi allow that information to affect their price setting for Pepsi products. Does this new edge running from Consumer Preferences to Pepsi-Price change your answer to parts i and ii given above? If so how does it change your answer in each case i and ii above? Suggest a strategy for finding unbiased and consistent estimates in each case (i and ii only).

3. Suppose you want to estimate the effect of climate change on the U.S. agricultural industry. Consider three different models that could be used to estimate how agricultural profits would respond to an increase in temperature: (i) an empirical production function (ii) a hedonic model, and (iii) a fixed effects model.

   a. Describe each empirical model in detail. At a minimum, you should answer the following questions for each model. What are the necessary assumptions? How would you determine whether those assumptions are plausible? What data would you need? What concerns would you have about the estimates from each model?

   b. Describe an alternative estimation method. Include a detailed description of the empirical model, assumptions, and data. Explain how this model addresses the concerns you listed for the models in part (a).

4. Consider an iid sample $\{Y_i, X_i\}_{i=1}^n$, where X_i is a K-dimensional covariates. Suppose the population model is given by

$$Y_i = X_i\beta + e_i, i = 1, \ldots, n. \tag{1}$$

(a) Suppose the error term follows a normal distribution with density function

$$f(e) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-e^2/2\sigma^2).$$

Write out the log likelihood function of the Maximum Likelihood Estimator of model (1). Derive the MLE solution of $\hat{\beta}_{MLE}$. Show that it is equivalent to the Ordinary Least Squares (OLS) estimator $\hat{\beta}_{OLS}$.

Suggested Answer: Ignoring a constant, the log likelihood is given by

$$L = \sum_i \{-\log\sigma - (Y_i - X_i\beta)^2/2\sigma^2\}$$

Maximizing L with respect to β is equivalent to minimizing sum of residual squares with respect to β; i.e., the OLS estimate.

(b) Instead of the normality assumption on the error term given above, suppose that $E[e_i|X_i] = 0$, $E[X_i'X_i]$ is full-ranked, and $\text{Var}[e_i|X_i] = g(X_i)$, where $g : \mathbb{R}^K \to \mathbb{R}^+$ is an **unknown** non-constant function. How would you estimate β? Denote your new estimator by $\tilde{\beta}$. Is $\tilde{\beta}$ different from $\hat{\beta}_{MLE}$? How would you estimate the variance covariance of $\tilde{\beta}$?

Suggested Answer: Now the error terms are heteroskedastic, the MLE with a constant variance does not correctly characterize the error distribution. Without explicit knowledge about the variance structure, OLS is recommended for the estimation of β. Thus $\tilde{\beta} = \hat{\beta} = \hat{\beta}_{OLS}$. However, the inference should be different as the OLS covariance matrix (under the assumption of homoskedasticity) is not consistent. Instead, one should use White's sandwich covariance, which is robust against unknown heteroskedascitity.

(c) Assume that the conditions in part (b) hold. Now instead of a simple I.I.D.

sample, we have a panel data $\{Y_{it}, X_{it}\}$ for $i = 1, \ldots, N$ and $t = 1, \ldots, T$ for the same model. Is your proposed estimator in part (b) asymptotically efficient? If not, propose an alternative estimator that is asymptotically efficient.

Suggested Answer: The proposed estimator in part (b) is not efficient as it does not tackle the unknown heteroskedasticity. Now with panel data, the unknown variance can be estimated via

$$\hat{g} = 1/T \sum_{i=1}^{n} \hat{e}_i \hat{e}_i^T$$

where $\hat{e}_i = (e_{i1}, \ldots, \hat{e}_{iT})^T$ are the residuals from some consistent estimator. One can then construct a feasible GLS estimator using the estimated covariance matrix. This estimator is asymptotically efficient.

5. A sample of N respondents have to choose among J available alternatives during T choice situations. The utility person n derives from choosing alternative j on choice situation t is given by $U_{njt} = \beta_n' x_{njt} + \varepsilon_{njt}$. The density for β is denoted as $f(\beta|\theta)$ where θ represents the distribution parameters. The unconditional choice probabilities of person n choosing i on occasion t are:

$$P_{ni} = \int \frac{e^{\beta' x_{ni}}}{\sum_{j=1}^{J} e^{\beta' x_{nj}}} f(\beta|\theta)\, d\beta$$

In order to get credit, please make sure you provide an explanation to support your answers.

(a) Propose a way to estimate P_{ni}

(b) Is the proposed estimator of P_{ni} unbiased?

(c) In terms of the properties of P_{ni}, what is the range of values P_{ni} can take? How can the $Var(P_{ni})$ be reduced?

(d) What would be the log-likelihood function to estimate the parameters associated with P_{ni}?

(e) Is the log-likelihood estimator in (d) unbiased? Is it consistent?

6. An Almost Ideal Demand (AIDS) model of Deaton and Muellbauer (1980) has been widely used in the applied literature due to its attractive properties and ease of estimation. The Exam Affine Stone Index (EASI) demand model developed by Lewbel and Pendakur (2009) provides valuable advantages over AIDS model.

   a. Noting that ***p'w*** is the definition of the Stone log price index, the EASI class of cost functions is defined as y equals to an affine transformation of Stone index deflated log nominal expenditures, ***x – p'w***. The resulting EASI implicit Marshallian demand functions have six properties. Please, list at least three of these properties and discuss their similarities and advantages over AIDS.

   b. Provide the complete list of properties required for the EASI cost function

$$C(\mathbf{p}, u, \mathbf{z}, \boldsymbol{\varepsilon}) = u + \mathbf{p}'\left[\sum_{r=0}^{5} \mathbf{b}_r u^r + \mathbf{Cz} + \mathbf{Dz}u\right] + \frac{1}{2}\sum_{l=0}^{L} z_l \mathbf{p}'\mathbf{A}_l\mathbf{p} + \frac{1}{2}\mathbf{p}'\mathbf{Bp}u + \mathbf{p}'\boldsymbol{\varepsilon}.$$

   to satisfy cost function regularity.

   c. The EASI implicit Marshallian budget shares are given by:

$$\mathbf{w} = \sum_{r=0}^{5} \mathbf{b}_r y^r + \mathbf{Cz} + \mathbf{Dz}y + \sum_{l=0}^{L} z_l \mathbf{A}_l \mathbf{p} + \mathbf{Bp}y + \boldsymbol{\varepsilon},$$

   where *y* is a function of observables, *x, p, z,* and the log Stone index *p'w*. Discuss the estimators that can be used to estimate this model.