PhD Qualifier Examination

Department of Agricultural Economics

May 31, 2016

Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam. Show all your work. If necessary, use math, graphical analysis and provide definitions of key concepts.

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4.1) at the top of each page.
- Write on only one side of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages in order.

GOOD LUCK!

1. (15 points) Suppose we are interested in estimating a statistical model of crop yield $Y_{i,t}$ as a function of average growing season temperature $F_{i,t}$ and precipitation $P_{i,t}$, where $i = 1, \dots, N$ denote all U.S. counties and $t = 1, \dots, T$ denote year, spanning 1950 till 2015. Consider the following model

$$Y_{i,t} = \beta_0 + g_1(t) + g_2(F_{i,t}, P_{i,t}) + c_i + u_{i,t},$$

where g_1 and g_2 are unknown functions, c_i is county specific individual effect invariant over time, and $u_{i,t}$ is an error term with $E[u_{i,t}|t, F_{i,t}, P_{i,t}, c_i] = 0$.

   (a) $g_1(t)$ is a function of time that captures advances in technology, which generally trends up during the sample period. Propose a functional form for $g_1(t)$ that allows a flexible time trend. Propose a test concerning the hypothesis of a constant linear time trend.

   (b) It has been well documented that crop yield depends on temperature and precipitation. (i) Suggest a functional form that allows for nonlinear temperature and precipitation effects and interaction between temperature and precipitation. (ii) Heat resistance of crop yield has improved consistently during the last few decades. Suggest a functional form that allows for time varying effects of temperature. Propose a test for the hypothesis that the temperature effect is constant over time.

   (c) To account for the individual effects, one can use a fixed effects estimator or a random effects estimator. Which would you recommend? Explain your answer.

   (d) Potential heteroskedasticity in the error terms also has been discussed in the crop yield literature. One possibility is the following form:

$$\text{var}(u_{i,t}) = \sigma^2 Y_{i,t}^a,$$

   where $a \geq 0$. (i) Propose a test for the hypothesis $H_0 : a = 0$. (ii) If this hypothesis is rejected, propose an asymptotically efficient estimator that accounts for the heterodasticity.

2. (20 points) Consider the demand function for a product Q manufactured by a firm called CSI, Inc. This demand function depends on the price charged P and the level of advertising A for product Q as follows: $Q = 5,000 - 10P + 40A + PA - 0.8A^2 - 0.5P^2$. The data used to arrive at the ordinary least squares (OLS) parameter estimates were monthly time-series data from January 2010 to December 2015, a total of 72 observations. The R^2 metric is equal to 0.93 and the standard error associated with this regression analysis is equal to 6.

   (a) Calculate the adjusted R^2 statistic, the error sum of squares (SSE), and the accompanying F-statistic. Specify the degrees-of-freedom associated with this F-statistic.

   (b) How would you ascertain whether or not each of the explanatory variables (P, A, PA, A^2, and P^2) were key factors associated with the demand for product Q?

   (c) Given the fact that time-series are used in this analysis, how would you determine if it were necessary to correct for autocorrelation or serial correlation?

   (d) If autocorrelation were present, what procedure would you recommend to estimate the parameters of this demand function?

   (e) Given this demand specification, provide the expression for the change in Q due to a unit change in P as well as the expression for the change in Q due to a unit change in A. (These respective expressions should be in terms of P and A).

(f) Suppose that CSI, Inc. decided not to advertise (that is, $A = 0$), and suppose that CSI, Inc. wanted to charge a price of \$30 for Q. What is the value of the own-price elasticity of demand under these conditions? Characterize this own-price elasticity of demand.

(g) What is the optimal price and advertising combination, assuming CSI, Inc. wishes to maximize the amount of product Q to be manufactured?

(h) What is the maximum level of Q to be manufactured given this demand relationship and what is the maximum level of sales revenue?

3. (15 points) Consider a pure exchange economy with L commodities and n consumers, $i = 1, ..., n$, each having initial endowment vector $w_i \in \mathbb{R}^L_{++}$ and preference orderings $\succcurlyeq_i$ that can be represented by a continuous utility function $u_i : R^L_+ \to R$ which is assumed to be strictly monotone and strictly concave.

   (a) Using the above notation, formally define weak and Pareto efficiency, respectively.

   (b) Is every weak Pareto efficient allocation a Pareto efficient allocation under these conditions? If yes, prove it; otherwise give a counterexample.

   (c) State the first welfare theorem and prove it.

   (d) Characterize the sets in (a) in terms of an expression involving a summation using utilities. Justify your answer.

   (e) Briefly discuss the economic significance of the first and second welfare theorems.

4. (15 points) Consider a game with two players ($i = 1, 2$) with the following action and payoff space:

	b_1		b_2		b_3	
a_1	10	10	2	12	0	13
a_2	12	2	5	5	0	0
a_3	13	0	0	0	1	1

   (a) Find the set of rationalizable strategies for each player.

   (b) Find all Nash equilibria of the stage game.

   (c) Consider a game, in which the two players play the simultaneous move game twice. Players are impatient and discount the future according to a common discount factor δ. What is the lowest discount factor that can support playing (a_1, b_1) in the first period as an SPNE of this repeated game? Carefully define the strategies corresponding to this SPNE.

5. (20 points) Consider an Expected Utility maximizer, on the set of money lotteries, whose utility index is:

$$u(x) \equiv -e^{-ax},$$

where $a > 0$.

   (a) What function represents the preferences of the agent described above?

   (b) Define what is meant by a risk averse agent.

(c) Is the agent described above risk averse? (prove it if so, or disprove it if not).

(d) Imagine that this agent has an initial wealth $w > 0$. Let F be the lottery that multiplies the agent's wealth by a factor $k > 0$ with probability α and gives the agent 0 dollars with probability $(1 - \alpha)$.

i. Show that for any given $k > 1$ there is $0 < \alpha^* < 1$ such that the agent prefers lottery F to keeping her wealth for sure whenever $\alpha > \alpha^*$.

ii. Now, show that for any k and any $0 < \alpha < 1$, if the agent's initial wealth is sufficiently large, the agent prefers to keep her wealth for sure instead of lottery F.

iii. Show that this agent's coefficient of absolute risk aversion is constant.

iv. Is there any contradiction between the results above, i.e., why is it possible that the agent accepts a gamble that she would reject if she were wealthier and nevertheless she is not becoming more risk averse?

6. (15 points) Consider an exchange economy with three agents $N \equiv \{1, 2, 3\}$. Denote generic agents by i, j, k. The profile of endowments is $\Omega \equiv (\Omega^i)_{i \in N} \in \mathbb{R}_+^{L \times N}$. A (consumption-budget) allocation is a triple $(x, p, \omega) \in \mathbb{R}_+^{L \times N} \times \mathbb{R}_+^L \times \mathbb{R}_+^{L \times N}$, where $x \equiv (x^i)_{i \in N} \in \mathbb{R}_+^{L \times N}$ is a list of consumption bundles, $p \in \mathbb{R}_+^L$ is a price vector, and $\omega \in \mathbb{R}_+^{L \times N}$ is a possibly redistributed income profile. An allocation is feasible if $\sum_i x^i = \sum_i \Omega^i$ and $\sum_i \omega^i = \sum_i \Omega^i$. Different from a standard general equilibrium economy, agents here have preferences on the set of allocations, not on the consumption set. Agent i's preferences are represented by the utility function V^i that for each $(x, p, \omega) \in \mathbb{R}_+^{L \times N} \times \mathbb{R}_+^L \times \mathbb{R}_+^{L \times N}$ assigns the value:

$$V_i(x, p, \omega) \equiv u_i(x^i) - \frac{a_i}{2} \sum_{i \neq j} \max\{v_i(p, \omega^j) - v_i(p, \omega^i), 0\} - \frac{c_i}{2} \sum_{i \neq j} \max\{v_i(p, \omega^i) - v_i(p, \omega^j), 0\},$$

where $c_i \in (0, 1)$, $a_i > c_i$, u_i is a function defined on $\mathbb{R}_+^L$ and v_i is its indirect value function, that is $v_i(p, \omega^j) \equiv \max\{u_i(y) : p \cdot y \leq p \cdot \omega^j, y \geq 0\}$. A competitive equilibrium here is a feasible allocation (x, p, Ω) such that each agent, say i, maximizes her V_i preferences in her budget set at the given prices, i.e., chooses x_i in $B(p, p \cdot \Omega^i)$ maximizing $V(x_i, x_{-i}, p, \Omega)$.

(Some intuition of what is going on here: agents are not only concerned about their consumption, but also about the equitability of the opportunities among agents. The agent loses some welfare both when other agents have "better" and/or "worse" budget sets.)

(a) What does WARP stand for?

(b) State WARP.

(c) Imagine that we observe agent i's choices in all possible budget sets $B(p, p \cdot \Omega^i)$ when we keep fixed the endowment profile. Let $(\mathcal{B}, C^i)$ be the corresponding choice structure. Does it satisfy WARP? (provide an argument supporting your answer)

(d) Suppose now that each u_i is strictly increasing. Show that a competitive equilibrium in which all agents have the same income is Pareto efficient. More precisely. Let (x, p, Ω) be a competitive equilibrium at which for any two agents i, j, $p \cdot \Omega^i = p \cdot \Omega^j$. Show that there is no other feasible allocation (y, p', ω) such that for each agent $i \in N$, $V_i(y, p', \omega) \geq V_i(x, p, \Omega)$ and for some $j \in N$, $V_j(y, p', \omega) \geq V_j(x, p, \Omega)$.