PhD Qualifier Examination

Department of Agricultural Economics

May 29, 2015

Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam.

If an answer requires complicated mathematical calculations, students will be given full credit if they simply write down the function that could have been typed into a calculator.

Important procedural instructions:

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4.1) at the top of each page.
- Write on only one side of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages in order.

GOOD LUCK!

1. **(15 points)** Consider the following model:

$$Y_i = \beta_0 + \beta_1 X_i + h^2(X_i)u_i,$$

where X_i is a scalar, u_i is a random error with $E[u_i|X_i] = 0$ and $E[u_i^2|X_i] = \sigma^2 < \infty, \forall i$

(a) Suppose the Ordinary Least Squares (OLS) estimator is used to estimate this model. Discuss the statistical properties of the OLS estimator.

(b) Propose a test on the hypothesis of homoscedasticity of the error terms.

(c) Suppose that u_i are known to follow a normal distribution and one uses a maximum likelihood estimator under the (false) assumption of I.I.D. errors. Propose a likelihood based test for the hypothesis of homoscedasticity.

(d) Suppose it is known that $h(X_i) = \exp(X_i\theta)$. Propose an estimator for the heteroscedastic model (i.e., an estimator that estimates all parameters $(\beta_0, \beta_1, \theta)$ under the right functional form).

2. **(15 points)** Suppose that the structural specification for a production function is given by:

$$lnq_i = \beta_0 + \beta_1 lnX_{1i} + \beta_2 lnX_{2i} + \beta_3 lnX_{3i} + u_i,$$

Where q_i corresponds to levels of output, and X_{1i}, X_{2i}, X_{3i} correspond to three levels of inputs respectively; i=1,2,…,60. The data set type is cross sectional.

(a) Augment the specification to account for potential regional differences among extension districts. You realize that there are 8 extension districts. You suspect that differences indeed exist among extension districts. How can you verify for potential regional differences?

(b) You wish to test for economies of scale. What would be the null hypothesis? How do you test it?

(c) You suspect that the output elasticity for input 1 equals the output elasticity for input 3. What would be the null hypothesis? How do you test it?

(d) You suspect that the product of the output elasticities equals 0.8. What would be the null hypothesis? How do you test it?

(e) You suspect that all the output elasticities are equal. What would be the null hypothesis? How do you test it?

3. **(15 points)** Consider the following hidden action model. A risk neutral principal hires an agent to put effort in a project with two possible profit outcomes: $\pi_H = 10$ and $\pi_L = 0$. The likelihood of each profit outcome depends on the agent's effort level. There are three possible effort levels: $E = \{e_1, e_2, e_3\}$ with $e_1 > e_2 > e_3$ and corresponding effort cost function $g(e_1) = \frac{5}{3}, g(e_2) = \frac{8}{5}, and\ g(e_3) = \frac{4}{3}$. The agent's utility function takes the form: $u_a = v(w) - g(e)$ where $v(w) = \sqrt{w}$ and his reservation payoff is given by $\bar{u}_a = 0$.

   (a) Suppose that effort is observable and the principal wants to implement effort e_i. Specify a wage contract that would implement e_i in this complete information game.
   (b) What effort level will the principal choose to implement when effort is observable? (show your work).
   (c) Suppose now that effort is unobservable. Is effort level e_3 implementable in equilibrium? If so, specify the optimal wage offer that would implement such effort level.
   (d) Given unobservable effort, is effort level e_2 implementable in equilibrium? If so, specify the optimal wage offer that would implement such effort level. (Hint: It is easier to express the IR and IC constraints as a function of the utility level in the two stages $v_L = \sqrt{w_L}$ and $v_H = \sqrt{w_H}$ rather than the wage level w_L and w_H in the two states).
   (e) Given unobservable effort, is effort level e_1 implementable in equilibrium? If so, specify the optimal wage offer that would implement such effort level.

4. **(20 points)** Consider the following game between a risk-neutral mechanic and a risk-neutral car owner. The mechanic can be one of two types. With probability $\frac{1}{5}$ the mechanic is good ($\theta = G$), and with probability $\frac{4}{5}$ the mechanic is bad ($\theta = B$). After learning his type, the mechanic has three options. He can suggest high cost repairs ($s_1 = H$), medium cost repairs ($s_1 = M$), or low cost repairs ($s_1 = L$). The car owner observes the mechanic's suggestion (but not his type) and chooses to accept ($s_2 = A$) or reject ($s_2 = R$) the suggestion. Let $\mu(G|s_1)$ denote the car owner's belief that the mechanic is good after observing suggestion s_1.The payoffs from each outcome are as follows:

   - If L is offered and rejected, both types receive 0 regardless of the mechanic's type.

- If L is offered and accepted, each party gets 1 if the mechanic is good. If the mechanic is bad, the mechanic gets 0.5 and the car owner gets -1.
- If M is offered and rejected, the mechanic gets 1 and the car owner gets -2 if the mechanic is good. If the mechanic is bad, the mechanic gets 0.5, and the car owner gets 1.5.
- If M is offered and accepted, each party gets 2 if the mechanic is good. If the mechanic is bad, the mechanic gets 1.5 and the car owner gets -1.5.
- If H is offered and rejected, the mechanic gets 1.5 and the car owner gets -3 if the mechanic is good. If the mechanic is bad, the mechanic gets 1, and the car owner gets 2.
- If H is offered and accepted, each party gets 3 if the mechanic is good. If the mechanic is bad, the mechanic gets 2 and the car owner gets -2.

(a) Draw a game tree representing this extensive form game. List all payoffs specifying the mechanic's payoffs first and the car owner's payoff second.

(b) Does a fully separating PBE exist? If so, list one of them. If not, explain why not.

(c) Does a fully pooling PBE exist? If so, list one of them. If not, explain why not.

(d) Specify a PBE in which the good mechanic plays $s_1 = M$ with probability 1 and the bad mechanic mixes.

5. **(20 points)** Let $e \equiv ((\succsim^i)_{i\in N}, (Y^j)_{j\in J}, (w^i)_{i\in N}, (\theta_j^i)_{i\in N, j\in J})$ be a private ownership economy. Assume that the consumption space of each agent is $\mathbb{R}_+^L$.

(a) Define what a feasible allocation for *e* is.

(b) Define competitive equilibrium for *e*.

(c) Define what a Pareto efficient allocation for *e* is.

(d) What is an exchange economy?

(e) State the First Welfare Theorem for *e*.

6. **(15 points)** Consider an agent who has wealth *w* and is an expected utility maximizer with utility index over amounts of money $u(z)$. There are *n* possible states of the world. State $i \in \{1, \dots, n\}$ is realized with probability α_i. There are *n* assets the agent may buy: one unit of asset $i \in \{1, \dots, n\}$ returns one unit of money in state *i* and zero in each other state (a unit of wealth that is not spent in any asset has zero return). The price of asset *i* is $p_i > 0$ and $p = (p_i)_{i\in N}$. Denote by z_i the amount of asset *i* the agent buys and $z \equiv (z_i)_{i=1}^n$.

(a) Write down the agent's optimal portfolio choice problem (utility maximization problem).

(b) Suppose that there are only two states $\{1,2\}$, the agent's utility index is $u(z) \equiv \log(z)$, and the agent has bought a portfolio $(z_1, z_2) \gg 0$. Before the state of the world is realized, the agent is approached by another agent who offers to buy z from her in exchange for a payment c that is independent of the state of the world (a risk-free asset). What is the minimum c, as a function of (α_1, α_2) and (z_1, z_2), for which the agent is willing-to-accept the deal? What type of preferences the agent has on portfolios (i.e., are they quasi-linear, homothetic, linear, monotone?)

(c) Suppose that there are only two states $\{1,2\}$ and there are K agents with identical utility indices $u_k(z) \equiv \log(z)$ for $k = 1, \dots, K$. Agent k has initial wealth w^k. Prove that the aggregate optimal portfolio is a function of prices and aggregate wealth $w \equiv w^1 + \cdots + w^K$.