

PhD Qualifier Examination

Department of Agricultural Economics

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Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam.

If an answer requires complicated mathematical calculations, students will be given full credit if they simply write down the function that could have been typed into a calculator.

Important procedural instructions:

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4.1) at the top of each page.
- Write on only **one side** of your paper and leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages in order.

GOOD LUCK!

1. **(15 Points)** An agent has preferences on the set of probability distributions on a finite set of prizes $Z \equiv \{z_1, \dots, z_n\}$. The space of these probability distributions is $\Delta(Z)$ and the generic probability function $\pi \equiv (\pi_i)_{i=1}^n$. The agent's preferences are represented by the utility function that assigns to each $\pi \in \Delta(Z)$ the value:

$$U(\pi) = \sqrt[3]{h + k \prod_{i=1}^n \alpha_i^{\pi_i}},$$

where $h, k, \alpha_1, \dots, \alpha_n$ are positive numbers and $\prod_{i=1}^n \alpha_i^{\pi_i}$ denotes the product $\alpha_1^{\pi_1} \dots \alpha_n^{\pi_n}$.

- (a) What are the axioms that characterize Expected Utility representation of preferences in von Neumann-Morgenstern theorem? Provide a formal statement of each axiom.
- (b) Which of the EU axioms are satisfied by the preference above?
2. **(15 Points)** Consider the simple linear model

$$Y_i = X_i \beta + u_i, i = 1, \dots, N,$$

where X_i is a scalar with $E[X_i] = 0$ and u_i 's are random errors with mean zero and finite variance σ^2 .

- (a) Derive an estimator of β . Express its expectation in terms of variance and/or covariance that involve X_i .
- (b) Derive the probabilistic limit of your estimator. State clearly the assumptions needed for your answer.
- (c) Derive the asymptotic variance of your estimator.
- (d) Suppose that $Y_i = \log Q_i$ and $X_i = \log P_i$, where Q and P denote consumption and price of beef respectively.
- i. Derive the price elasticity in this estimation.
 - ii. How does the elasticity vary with price in this model?
 - iii. Suppose that beef consumption is influenced by the price of chicken, which is a substitute to beef. It is also known that beef and chicken prices tend

to move in the same direction. Discuss the consequence on the estimation of own price elasticity due to omitting the price of chicken in this regression.

3. **(15 Points)** Consider an exchange economy with two consumers and two commodities x and y . Suppose $X_i = R_+^2$. Each consumer has an income of 1.

- (a) Consumer A's preferences are represented by the utility function

$$u_A(x_A, y_A) = x_A^\alpha \times y_A,$$

where $\alpha > 0$. Draw A's indifference curves. What is A's demand function? Graph his demand for x and y as a function of p . Are A's preferences convex? Monotone? Strictly convex? Strictly monotone? Continuous?

- (b) Consumer B's preferences are represented by the utility function

$$u_B(x_B, y_B) = (x_B)^2 + 2(y_B)^2.$$

Draw B's indifference curves. What is B's demand function? Graph his demand for x and y as a function of p . Are B's preferences convex? Monotone? Strictly convex? Strictly monotone? Continuous?

- (c) Suppose we have an economy with two consumers, both having preferences like consumer A's, and endowment equal to $(1, 2)$.
- Find all the Walrasian equilibria. Is it unique?
 - Find the set of core allocations in the Edgeworth Box.
 - Find the set of fair allocations in the Edgeworth Box.

4. **(15 Points)** Let $Y \subseteq \mathbb{R}^L$ be a production set.

- (a) State the definition of additivity of a production set.
- (b) Argue in one paragraph that the production set of a firm in an industry in which there is free entry is always additive.

- (c) A private ownership economy is a collection of agents N represented by preferences $(\succsim_i)_{i \in N}$ and firms $J = \{i, \dots, j\}$ represented by production sets $(Y^j)_{j \in J}$. Agent $i \in N$ has initial endowment of resources $w^i \in \mathbb{R}_+^L$ and for each firm $j \in J$, agent i owns a proportion θ_{ij} of firm j . A competitive equilibrium for the economy is a triple $(x \equiv (x^i)_{i \in N}, y \equiv (y^j)_{j \in J}, p \equiv (p_l)_{l=1}^L)$, where $x^i \in \mathbb{R}_+^L$ is agent i 's consumption, $y^j \in \mathbb{R}^L$ is firm j 's production vector, and p is a price vector, that satisfy among other conditions, that for each firm $j \in J$, y^j maximizes firm j 's profits at prices p . Argue that if firm j belongs to an industry in which there is free entry and possibility of inaction, then in any competitive equilibrium firm j 's profit is zero.

5. **(20 Points)** Consider a panel data model

$$Y_{it} = X_{it}\beta + c_i + u_{it}, \quad i = 1, \dots, N, \quad t = 1, \dots, T,$$

where $E[u_{it}|X_{i1}, \dots, X_{iT}, c_i] = 0$ and $E[c_i u_{it}] = 0$ for all i 's and t 's. Denote by $v_{it} = c_i + u_{it}$ and $v_i = (v_{i1}, \dots, v_{iT})'$. Define

$$\Sigma = E[v_i v_i'].$$

- (a) Propose a feasible generalized least squares estimator (FGLS) that utilizes an estimate of Σ . State the necessary steps to implement your proposed estimator.
- (b) Let $\text{Var}(c_i) = \sigma_c^2$. Further assume that

$$\text{Var}(u_{it}|X_{i1}, \dots, X_{iT}, c_i) = \sigma_u^2.$$

Propose a FGLS estimator under this additional assumption. State the necessary steps to implement your proposed estimator.

- (c) Discuss the relative advantages and limitations of the two estimators proposed above. Give your recommendations on how to select between these two estimators.
- (d) Suppose that $N = 500$ and $T = 10$. The results based on your proposed estimators

from Part (a) and (b) are considerably different. Which estimator would you recommend? Why?

6. **(20 Points)** Consider two identical risk neutral firms, $i = 1, 2$ that produce identical product at a constant marginal cost $c > 0$. There is a single consumer with unit demand for the good who is known to value the good at $v > c$. Suppose that the firms simultaneously announce prices $p_i \geq 0$. Upon observing the price, the consumer chooses whether to buy the good from one of the sellers. In case of a price tie, the consumer chooses one of the firms at random.

- (a) Do the firms have any strictly dominated strategies?
- (b) Do the firms have any weakly dominated strategies?
- (c) Solve for the pure strategy Nash equilibrium and argue that this equilibrium is unique.

Suppose now that the firms incur cost $q \in (0, v - c)$ to issue a price quote (for instance, a salesperson makes a costly phone call to the buyer). The firms simultaneously choose whether to give the buyer a costly price quote and upon observing the quoted prices, the buyer makes her purchasing decision.

- (d) Is there a pure strategy Nash equilibrium in this game.
- (e) Consider now a mixed strategy Nash equilibrium, in which each firm gives a price quote with probability $\alpha \in (0, 1)$. Conditional on making a price offer, each firm prices according to a continuous and atomless cdf $F(\cdot)$. Explain why in such mixed strategy equilibrium, a firm makes a price offer lower than $c + q$ and higher than v with zero probability.
- (f) Write the expected payoff of firm i from quoting a price p_i given that the other firm adheres to the mixed equilibrium strategy described in part e).
- (g) Solve for the equilibrium values of $F(p)$ and α assuming that the support of $F(\cdot)$ is the entire interval $[c + q, v]$.