PhD Qualifier Examination

Department of Agricultural Economics

May 29, 2013

Instructions

The exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam.

If an answer requires complicated mathematical calculations, students will be given full credit if they simply write down the function that could have been typed into a calculator.

Important procedural instructions:

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4.1) at the top of each page.
- Write on only **one side** of your paper and the leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages in order.

GOOD LUCK!

1. **(20 points)** You wish to estimate the following relationship using OLS:

$$y = x\beta + \varepsilon,$$

where x is a scalar (assume the data have been demeaned to remove the constant). However, your dataset does not include x. Rather, your dataset includes $x^* = x + \eta$. Thus, you use OLS to estimate:

$$y = x^*\delta + \nu.$$

(a) Suppose that $E[x^*\nu] = 0$. What is the probability limit of $\hat{\delta}_1$, the OLS estimate of δ in terms of x, y, ε and η.

(b) Compare the probability limit of $\hat{\delta}_1$ to the true value of β.

(c) Suppose that $E[x\nu] = 0$. What is the probability limit of $\hat{\delta}_2$, the OLS estimate of δ in terms of x, y, ε and η.

(d) Provide the necessary restrictions on the model parameters that would guarantee that the probability limit of $\hat{\delta}_1$ equals the probability limit of $\hat{\delta}_2$.

(e) Suppose your dataset also includes the variable z, such that $E[z\eta] = 0$, and you know that $E[x\eta] = 0$. Derive a method of moments estimator of β (denote this $\hat{\delta}_3$).

(f) What is necessary for $\hat{\delta}_3$ to be well-defined?

(g) Provide the necessary moment condition under which $\hat{\delta}_3$ is a consistent estimator of β.

2. **(15 points)** A buyer wishes to sell her used car. She values it at $r(\theta) = \theta - 1/2$, where $\theta \in \{1, 2\}$ denotes the quality of the car. Let α denote the probability of high quality and assume that $\alpha < 1/2$. Two identical risk neutral buyers $i = 1, 2$ with valuation θ are interested in purchasing it. They simultaneously offer bids p_1 and p_2 to the buyer, who subsequently decides whether to sell the car to one of the buyers or keep it. Suppose that she sells if she is indifferent about keeping the car and she flips a fair coin if she is indifferent between the two bids.

(a) Suppose that the buyers can observe the quality directly. Derive the Subgame perfect Nash equilibrium (SPNE) bids $p_1^*(\theta)$ and $p_2^*(\theta)$.

(b) What is the ex-ante expected value of the equilibrium social surplus?

(c) Suppose now that the quality of the car is private information for the seller. What are the Bayesian Nash equilibrium (BNE) bids p_1^* and p_2^*?

(d) What is the ex-ante expected value of the social surplus generated under the BNE in part (c)? How does it compare to the equilibrium in part (a)?

(e) Suppose now that before putting the car on the market, the seller can take it to a mechanic, who can certify its quality honestly for a fee $c < 1/2$. Let $q = \{0, 1, 2\}$ denotes the case of no certification, low quality or high quality certification. Which types, if any, get inspected in a Perfect Bayesian equilibrium (PBE) of this game?

(f) Derive the PBE belief of high quality $\mu^*(q)$ and bids $p_1^*(q), p_2^*(q)$.

3. **(15 points)** Elasticities play a major role in economic analyses.

(a) Provide a formal definition for any elasticity.

(b) Describe the usefulness of elasticities in economics and in managerial decision-making.

(c) Suppose we are concerned with the calculations of elasticities from the econometric specification for beef given below using quarterly time series data from 1996:1 to 2012:4.

$$\begin{aligned}\text{QBEEF}_t =& a_0 + a_1\text{PBEEF}_t + a_2\text{PPORK}_t + a_3\text{PCHICKEN}_t \\ &+ a_4\text{INCOME}_t + a_5 Q1_t + a_6 Q2_t + a_7 Q3_t + e_t,\end{aligned}$$

where

- QBEEF_t = per capita consumption of beef in quarter t;
- PBEEF_t = inflation-adjusted price of beef in quarter t;
- PPORK_t = inflation-adjusted price of pork in quarter t;
- PCHICKEN_t = inflation-adjusted price of chicken in quarter t;
- INCOME_t = inflation-adjusted per capita income in quarter t;
- $Q1_t, Q2_t, Q3_t$ refer to dummy variables for quarter 1, quarter 2, and quarter 3 respectively.

i. Provide the computational formula for the derivation of the own-price elasticity of beef. What is the expectation of the sign of this elasticity?

ii. Provide the computational formula for the derivation of the cross-price elasticities for beef with respect to the prices of pork and chicken? What are the expectations of the signs of these cross-price elasticities?

iii. How would you test whether or not seasonality is evident in per capita beef consumption? Please indicate the statistical test and the accompanying degrees-of-freedom.

(d) If the respective prices in the previous econometric specification were replaced by natural log transformation of the prices, how do the computational formulas for the own and cross-price elasticities change?

(e) Let the matrix of own-price, cross-price, and income elasticities be given as follows for two commodities.

Good 1	Good 2	Income
-0.6	0.1	?
?	?	?

Be aware the budget shares (w_i) for the two commodities are as follows: $w_1 = 0.4$ and $w_2 = 0.6$. The own-price elasticity of good 1 is -0.6, and the own-price elasticity of good 1 with respect to good 2 is 0.1.

i. Fill in the remaining entries of elasticities using restrictions from neo-classical demand theory.

ii. Are the goods in question inferior, normal, or luxury items?

iii. Are the goods in question substitutes, complements, or independent?

iv. Does this matrix of elasticities conform to the negativity condition?

4. **(20 points)** Consider an agent who has wealth w and is an expected utility maximizer with utility index over amounts of money $u(z)$. There are n possible states of the world. State $i \in \{1, ..., n\}$ is realized with probability α_i. There are n assets the agent may buy: one unit of asset $i \in \{1, ..., n\}$ returns one unit of money in state i and zero in each other state (a unit of wealth that is not spent in any asset has zero return). The price of asset i is $p_i > 0$. Denote by z_i the amount of asset i the agent buys and $z \equiv (z_i)_{i=1}^n$.

   (a) Write down the agent's optimal portfolio choice problem (utility maximization problem).

   (b) Suppose that there are only two states $\{1, 2\}$, the agent's utility index is $u(z) \equiv \log(z)$, and the agent has bought a portfolio (z_1, z_2). Before the state of the world is realized, the agent is approached by another agent who offers to buy z from her in exchange for a payment c that is independent of the state of the world (a risk-free asset). What is the minimum c, as a function of (α_1, α_2) and (z_1, z_2), for which the agent is willing to accept the deal ?

   (c) Suppose that there are only two states $\{1, 2\}$ and there are K agents with identical utility indices $u_k(z) \equiv \log(z)$ for $k = 1, \ldots, K$. Agent k has initial wealth w_k. Prove that the aggregate optimal portfolio is a function of prices and aggregate wealth $w \equiv w_1 + \cdots + w_K$.

   (d) Suppose that there are n states and there are k agents with identical utility indices $u_k(z)$ that are homogeneous of degree $r > 0$. Agent k has initial wealth w_k. Prove that the aggregate optimal portfolio is a function of prices and aggregate wealth $w \equiv w_1 + \cdots + w_k$.

5. **(15 points)** Suppose we observe iid data $\{X_i, Y_i\}, i = 1, \ldots, n$, where Y_i is a scalar and X_i is $K \times 1$ vector.

Consider the following model

$$Y_i = X_i\beta + h(X_i)e_i, \tag{1}$$

where e_i is an iid error term with $E(e_i|X_i) = 0$ and $\text{Var}(e_i|X_i) = 1$, and $h(X_i)$ is an unknown bounded function of X_i.

(a) Suppose we estimate model (1) with the OLS. Derive the probability limit of $\hat{\beta}$, the OLS estimate. Is this estimator consistent?

(b) Is the OLS estimate $\hat{\beta}$ asymptotically efficient? If your answer is negative, what additional condition(s) are needed to make the OLS estimate efficient? Provide a step-by-step description of your estimator.

(c) Suppose $h(X_i) = \sqrt{\exp(X_i\alpha)}$. Present an efficient estimator for model (1) under this condition.

6. **(15 points)** Consider an exchange economy with two agents and three commodities $X = (X^1, X^2, X^3)$. Suppose $X_i = R^3_+$,

$$u_1(x_1^1, x_1^2, x_1^3) = \min(x_1^1, x_1^2, x_1^3); w_1 = (1, 1, 1)$$
$$u_2(x_2^1, x_2^2, x_2^3) = \frac{1}{3}\ln x_2^1 + \frac{1}{2}\ln x_2^2 + \frac{1}{6}\ln x_2^3; w_2 = (1, 1, 1)$$

(a) Find the demand functions of these two agents.

(b) Find the aggregate excess demand functions of this economy,

(c) Find a competitive equilibrium allocation and price.

(d) Is the competitive equilibrium price unique? Why?

(e) Is the competitive equilibrium allocation Pareto efficient? Why?

(f) Define global stability for this economy. Is the equilibrium you found in part (c) globally stable? Why?