

Department of Agricultural Economics

PhD Qualifier Examination

May 2012

Instructions:

The exam consists of six questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam.

If an answer requires complicated mathematical calculations, students will be given full credit if they simply write down the function that could have been typed into a calculator.

Important procedural instructions:

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Put the question number and page number for that question at the top of each page.
- Write on only one side of your paper.
- Leave at least 1 inch margins on all sides.
- Turn in your final copy with all pages in order.

Good Luck!

1. (15 points) Consider an expected utility maximizer with utility index $u(y) = -e^{-\theta y}$ with $\theta > 0$, which is a function of ex post wealth y , and initial wealth $w > 0$. There is a risk free asset that has a return of 2 (if the agent invest x dollars, then she receives $2x$). There is also a risky asset with a random return of 1 with probability $\alpha > 0$ and 3 with probability $(1 - \alpha)$.
 - a. Set up the portfolio choice problem for the agent (denote the amount invested in the risky asset by x and the optimal portfolio by x^*).
 - b. Under what conditions is $x^* > 0$? Interpret.
 - c. Under what conditions is $x^* < w$? Interpret
 - d. Assume that the conditions guaranteeing that x^* is interior are satisfied. Calculate x^* .
 - e. How does x^* depend on θ ? Interpret.

2. (15 points) Let $\{X_i, Y_i\}, i = 1, 2, \dots, n$, be a sample of random bivariate i.i.d. data. Consider the following two scenarios:
 - (A) Let $u_i; i = 1, 2, \dots, n$, be i.i.d. random variables from the standard log normal distribution, and

$$Y_i = \exp(X_i \beta_1) \times u_i; i = 1, 2, \dots, n.$$
 - (B) Let $v_i; i = 1, 2, \dots, n$, be i.i.d. random variables from a non-degenerate distribution with mean zero and finite variance, and

$$Y_i = \exp(X_i \beta_2) + v_i; i = 1, 2, \dots, n.$$
 - a. Present estimators for β_1 in Scenario (A) and for β_2 in Scenario (B) respectively. Describe the necessary steps to implement your estimators.
 - b. Denote your proposed estimators by $\hat{\beta}_1$ and $\hat{\beta}_2$ respectively. Present estimators for the variance of $\hat{\beta}_1$ and $\hat{\beta}_2$ respectively. Describe the necessary steps to implement your estimators.
 - c. Suppose it is known that $E[Y | X = x] = \exp(x\beta)$. Discuss how one can choose between $\hat{\beta}_1$ and $\hat{\beta}_2$.

3. (20 points) Consider a patent race game with 2 firms (A and B). Each of the firms chooses simultaneously their R&D expenditure x_i ($i=A,B$). The firms are risk neutral and there is no discounting. Innovation occurs for firm i at time $T_i(x_i)$ where

$\frac{\partial T_i(x_i)}{\partial x_i} < 0$. The value of the patent V is identical for both firms and common

knowledge. The value is accrued to the firm that comes up with the innovation first. If both firms innovate simultaneously, each one of them has an equal chance of getting the patent.

- Write down firm i 's payoff function $\Pi_i(V, x_i, x_j)$ $i = A, B$.
- Does this game have any pure strategy Nash Equilibria? Either specify such equilibrium or prove that it does not exist.
- Find a symmetric mixed strategy Nash Equilibrium of the game.
- Suppose that the time of the innovation's arrival for firm i is given by $T_i(x_i) = \frac{1}{x_i}$

Let $T(x_A, x_B)$ denote the time of arrival of the innovation for a given choice of investment by the two firms. Write down an equation for $T(x_A, x_B)$. Using the mixed strategy equilibrium that you derived in part c, derive the expected equilibrium time of arrival of the innovative idea, $ET^*(V)$. How does $ET^*(V)$ depend on the patent value V ? Explain the economic intuition behind this relationship.

4. You wish to estimate the following relationship using OLS:

$$y_i = x_i \beta + \varepsilon_i$$

where x_i is a scalar for each individual, $i=1, \dots, n$. Assume the data have been demeaned to remove the constant.

Your data set does not include observations of the variable y_i . Instead, your dataset includes $y_i^* = y_i + v_i$.

Thus, you use OLS to estimate: $y_i^* = x_i \gamma + \varepsilon_i$.

- Derive the probability limit of $\hat{\gamma}$, the OLS estimate of γ . What are the least restrictive conditions required for $\hat{\gamma}$ to be a consistent estimate of β ?
- Suppose that the least restrictive conditions required for $\hat{\gamma}$ to be a consistent estimate of β are satisfied. Derive the asymptotic distribution of $\hat{\gamma}$. Compare the estimated standard error of $\hat{\gamma}$ with the standard error of $\hat{\beta}$? (State clearly any assumptions needed for your answer.)

For the rest of this question, suppose another researcher has a different data set that includes the variable y_i but does not include the x_i . Instead, this data set contains the variable $x_i^* = x_i + \eta_i$.

Thus, this researcher uses OLS to estimate: $y_i = x_i^* \delta + \varepsilon_i$.

- Derive the probability limit of $\hat{\delta}$, the OLS estimate of δ . (State clearly any assumptions needed for your answer.)
- Suppose that $E[x\eta]=0$. Compare the probability limit of $\hat{\delta}$, to the true value of β .
- Suppose that $E[x\eta]>0$ and $\beta>0$. Compare the probability limit of $\hat{\delta}$, to the true value of β .
- What are the least restrictive conditions required for $\hat{\delta}$ to be a consistent estimator of β ?

5. (15 points) Consider a pure-exchange economy with two consumers, **A** and **B** and with two commodities, x^1 and x^2 . Utility function and endowments for the two consumers are given by

$$u_A = \min[x_A^1, x_A^2], \quad \omega_A = (3, 3)$$

$$u_B = \min[x_B^1, x_B^2], \quad \omega_B = (3, 3)$$

- In a graph, depict the Pareto set and contract curve.
- Find all competitive equilibria.
- Are all competitive equilibria Pareto optimal? Explain your answer.
- Verify that the second welfare theorem applies.
- Now suppose that initial endowments are changed to $\omega_A = (4, 2)$ and $\omega_B = (1, 4)$. Answer (a), (b), and (c).

6. (15 points) Consider the preference on \mathbb{R}_+^2 represented by

$$U(x, y) = (x+a)^\alpha (y+b)^{1-\alpha},$$

where $a > 0$ and $b > 0$.

- Illustrate income expansion paths for this preference.
- Provide an expression for the demand function associated with this preference when consumption is interior.
- A good is said to be a luxury good at (p, w) if its income elasticity is greater than or equal to 1. Provide a characterization in terms of $\frac{\partial U}{\partial x}(0, 0)$ and $\frac{\partial U}{\partial y}(0, 0)$ of the combinations (p, w) for which a good, say x , is a luxury good when its consumption is interior. Interpret.