Department of Agricultural Economics

PhD Qualifier Examination

May 2010

Instructions:

The exam consists of six questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear in your answer as possible. You have four hours to complete the exam.

If an answer requires complicated mathematical calculations, students will be given full credit if they simply write down the function that could have been typed into a calculator.

Important procedural instructions:

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4.1) at the top of each page.
- Write on only **one side** of your paper and the leave at least 1” margins on all sides.
- Turn in your final copy with all pages in order

Good Luck!

1. (15 points)
Let $\succsim$ be the rational preference relation on $\mathbb{R}^2_+$ represented by the following utility function: for each $(x_1, x_2) \in \mathbb{R}^2_+$,
$$[u(x_1, x_2) \equiv \min\{x_1, x_2^2\}].$$

   a. Draw three different indifference curves.

   b. Let $x^{\succsim}$ be the demand function associated with $\succsim$. Let a price $p \in \mathbb{R}^2_{++}$ and wealth $w \in \mathbb{R}_+$. Provide a formal definition of $x^{\succsim}$.
   Calculate $x^{\succsim}(p,w) \equiv (x_1^{\succsim}(p,w), x_2^{\succsim}(p,w))$.

   c. What is the value of the Slutsky matrix associated with $x^{\succsim}$ at some arbitrary $p \in \mathbb{R}^2_{++}$ and $w \in \mathbb{R}_+$.

2. (15 points)
Let $(X_1, \ldots, X_T)$ be independently and identically distributed random variables from a distribution with mean μ and finite variance σ^2

   a. Define
   $$\hat{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T}(X_t - \hat{\mu})^2$$
   where $\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} X_t$. Show that $\hat{\sigma}^2$ is a biased but consistent estimator of σ^2.

   b. Suppose that $\mu = 0$, and for $t = 2, \ldots, n$,
   $$Y_t = \rho Y_{t-1} + X_t, \quad |\rho| < 1.$$
   Derive the mean and variance of Y_t.

   c. Calculate the covariance between Y_t and Y_s $t \neq s$

   d. Design a test for the hypothesis
   $$H_o : \rho = 0.$$
   Discuss properties of the proposed test statistic under the null hypothesis.

3. (20 points)
Suppose that there are two assets, a safe asset with a return of 1 dollar per dollar invested and a risky asset with a random return of z dollars per dollar invested. The random return z has the following probability distribution: z=**0** with probability α, z=**2** with probability β, and z=**1** with probability $1-\alpha-\beta$. A **risk neutral** individual, agent A, has an initial wealth w to invest in these two assets. Let x be the part of agent A's wealth invested in the risky asset. The amount invested in the safe asset is $w-x$. Thus, for any realization the random return z, agent A's portfolio $(x, w-x)$ pays $xz+(w-x)$.

a. What is agent A's optimal portfolio, i.e., find x in terms of α and β. When does agent A invest in the risky asset? Provide some intuition for your answer.

b. Suppose now that the agent must invest through a financial manager. There are two managers, M_1 and M_2, each of whom is paid as a portion $\varphi \in (0,1)$, of the agent's return. For instance, if M_1 is managing y_1 dollars and invests x_1 in the risky asset and $y_1 - x_1$ in the safe asset, M_1 is paid $\pi_1 = \varphi\left[x_1 z + \left(y_1 - x_1\right)\right]$ and agent A gets $(1-\varphi)(x_1 z + y_1 - x_1)$.

   i. Suppose that M_1 is an expected utility maximizer with von Neumann-Morgenstern (vNM) utility index: $u_1(\pi_1) \equiv \ln \pi_1$. If agent A gives y_1 dollars to M_1, what is the managers' optimal portfolio? That is, calculate x_1 in terms of α and β and specify when x_1=0 and x_1=y_1.

   ii. Suppose that M_2 is an expected utility maximizer with vNM utility index: $u_2(\pi_2) \equiv \sqrt{\pi_2}$. If agent A gives y_2 dollars to M_2, what is the managers' optimal portfolio? That is, calculate x_2 in terms of α and β and specify when x_2=0 and x_2=y_2.

   iii. Characterize agent A's optimal portfolio, i.e., find the amounts y_1 and y_2 that agent A gives to M_1 and M_2, respectively as functions of α and β. A graph may be helpful in presenting your results.

4. (15 points)
Consider a general binary response model $Pr\left(y_i = 1 \mid x_i\right) = \Phi\left(x_i\beta + \gamma\left(x_i\beta\right)^2\right)$, where Φ is the CDF of the standard normal. Suppose that we want to test the hypothesis

$$H_o : \gamma = 0 .$$

a. Construct the Wald test statistic for H_o

b. Construct the LM test statistic for H_o

c. Construct the likelihood ratio test statistic for H_o

d. Discuss the relationship between the three tests. Which test would you recommend for the model in consideration? Provide some reasons of your recommendation.

5. (20 points)
Consider a two-person two-commodity economy with

$$X_i = \left\{x_i \in \mathbb{R}_+^2 : x_i^1 + x_i^2 \geq 1\right\}$$
$$w_1 = (1,0)$$
$$w_2 = (1,2)$$
$$u_1 = x_1^1 + 2x_1^2$$
$$u_2 = \ln\left(n \cdot x_2^1\right) + \ln\left(nx_2^2\right)$$

a. Draw the Edgeworth Box with the consumption set, indifference curves, and the initial endowment point clearly marked.

b. Calculate demand functions for agents 1 and 2.

c. Find the set of Pareto optimal allocation and draw them in the Edgeworth Box.

d. Does a competitive equilibrium exist? If so, is it a Pareto optimal? If not, which of the standard assumptions for the existence theorem is violated?

e. Change the consumption sets to
$x_i = \mathbb{R}_+^2, i = 1, 2$
Does the change make a difference in your answer to part d above?

6. (15 points)
Consider the Cournot duopoly model when goods are differentiated. Player I produces widgets at unit cost c_1, but player II produces wowsers at unit cost c_2. If q_1 widgets and q_2 wowsers are produced, the respective prices for the two goods are determined by the demand equations $p_1 = M - 2q_1 - q_2$ and $p_2 = M - q_1 - 2q_2$. Adapt Cournot's duopoly model to this new situation, and find:

a. Derive the equations for the players' reaction curves.

b. At the equilibrium, what are the quantities produced and market prices.

c. Is the equilibrium identified in part b a Nash equilibrium? Explain.

d. Now suppose that M=30, c_1=c_2=0 and the firms' technology is such that they can produce only at two levels, $M/5$ or $M/6$. Using a normal form representation, identify the strategies for each player and the resulting equilibrium.