Department of Agricultural Economics

PhD Qualifier Examination

May 2009

<u>Instructions:</u>

The exam consists of six questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear in your answer as possible. You have four hours to complete the exam.

If an answer requires complicated mathematical calculations, students will be given full credit if they simply write down the function that could have been typed into a calculator.

Important procedural instructions:

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.

- Also, put the question number and answer page number (e.g. 4.1) at the top of each page.

- Write on only **one side** of your paper and the leave at least 1" margins on all sides.

- Turn in your final copy with all pages in order

Good Luck!

1. (20 points) Consider the following utility function

$$u(x_1, x_2) = x_1 - \frac{\beta}{2(x_2)^2}$$

where $\beta > 0$:

a. Set up the Lagrangian and write out the Kuhn-Tucker conditions for the utility maximization problem (UMP) with constraints $\omega \geq p_1 x_1 + p_2 x_2$, $x_1 \geq 0$ and $x_2 \geq 0$ with ω, p_1, $p_2 > 0$. Note that you do not need to solve anything for part a.

b. Under what conditions for β do we have a corner solution? Your answer should not include the Lagrange multiplier, λ.

For the remaining items assume there is an interior solution (i.e., ignore corner solutions in your answers).

c. Derive the demand function $x(p, w)$ for the case of an interior solution. You may use any method you choose.

d. Derive the indirect utility function $v(p, \omega)$.

e. Derive the Hicksian demand $h(p, u)$ and expenditure $e(p, u)$ functions using the duality identities.

2. Consider a system of two equations

$$y_1 = \alpha_1 y_2 + x_1' \beta + u_1,$$
$$y_2 = x' \eta_2 + u_2,$$

where x is a vector of exogenous variables, $x_1 \in x$, and $E[u_1 \mid u_2] \neq 0$.

a. Show that the OLS estimate of the first equation is inconsistent.

b. Let $\hat{\eta}_2$ be the OLS estimate of the second equation, and denote $\hat{u}_2 = y_2 - x' \hat{\eta}_2$. Assume that the linear projection $L(u_1 \mid x, u_2) = L(u_1 \mid u_2)$. Show that the OLS regression of y_1 on y_2, x_1 and $\hat{u}_2$ is consistent.

3. (20 points) For all parts of this question, consider an individual who has utility preferences over levels of ex post *wealth*, x, given by

$$u(x) = \alpha x^2, \text{ with } \alpha > 0.$$

a. What is his certainty equivalent payoff for a gamble to gain \$100 with probability $\frac{2}{3}$ and lose \$50 with probability $\frac{1}{3}$? Assume his current wealth is w.

b. Suppose instead the utility function is defined over the level of income. What is the certainty equivalent payoff for a gamble to gain \$30 with probability $\frac{1}{3}$ and gain \$6 with probability $\frac{2}{3}$?

c. When choosing between two gambles G_1 and G_2, which gamble would this individual prefer if G_1 first order stochastically dominated G_2? What if neither gamble first order stochastically dominates the other, but G_1 second order stochastically dominates G_2? Explain your answers.

The final two items concern insurance. Suppose there are two possible states of the world. State 1 occurs with probability $\frac{1}{2}$; state 2 occurs with probability $\frac{1}{2}$. The individual will have the final wealth of \$105 in state 1 and the final wealth of \$15 in state 2. The individual's utility function in either state is as given above.

The following insurance contract is offered to this individual. He agrees to pay z_1 if state 1 occurs and in return will receive z_2 if state 2 occurs. The contract is set up so as to eliminate any uncertainty, i.e. $\$105 - z_1 = \$15 + z_2$.

d. What is the highest level z_1 the individual would agree to forgo in state 1 and still prefer to have insurance? What is the expected loss/gain for the individual on such policy?

e. Suppose the individual above (i.e. with the same preferences, initial wealth, and facing the same gamble) was offered but rejected the following insurance policy: pay \$10 if state 1 occurs, receive \$40 if state 2 occurs. This result could be explained by subjective utility theory (SEU). What is the highest subjective probability of state 2 occurring that could explain this result?

4. (15 points) Consider a Cournot Oligopoly Model with a linear inverse demand function given by $p = M - q$, where p is market price, q is quantity of widgets, and M is a constant. Each widget costs a firm c to produce.

   a. Suppose that each firm has to pay a fixed cost of F regardless of the quantity it produces in order to enter the widget industry. Show that no firm's behavior is modified, provided that the fixed cost F is less than each player's equilibrium profit. Find the threshold for F. Compute equilibrium profits with fixed entry cost.

   b. If the fixed cost exceeds the equilibrium profit with n firms, then at least one firm would have been better off if it had not entered the widget industry. Assuming there are no barriers to entry other than payment of the fixed entry cost of F, determine the number of firms that will end up producing widgets.

   c. What happens to the number of firms, market price, firm output, firm profits, and consumer surplus as F goes to 0?

5. Suppose $y = a + bx_1 + cx_2 + u$, where a, b, c are constants, x_1 and x_2 are bivariate normal random variables with mean $[2, 4]$ and variance-covariance matrix

$$\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix},$$

and u is a uniform random variable defined on $[-1, 1]$.

a. Calculate $E[y]$ and $\operatorname{var}(y)$.

b. Calculate $E[y \mid x]$ and $\operatorname{var}(y \mid x)$.

c. Let $\{Y_i, X_{1,i}, X_{2,i}, U_i\}_{i=1}^{N}$ be an i.i.d. sample from the above distributions, and $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$. Calculate $E[\bar{Y} \mid X]$ and $\operatorname{var}(\bar{Y} \mid X)$.

6. (20 points) Consider an exchange economy with two agents and two commodities. Suppose $X = R^{2+}$,

$$U_1(x_{11}, x_{12}) = \min\{3x_{11}, 2x_{12}\}; \ w_1 = (1,1)$$

$$U_2(x_{21}, x_{22}) = \frac{1}{2}\ln x_{21} + \frac{1}{3}\ln x_{22}; \ w_2 = (1,1)$$

where x_{ij} is the consumption by individual i of good j, and w_i is the vector of initial endowments of the goods.

a. Find the demand functions of these two agents.

b. Find the aggregate excess demand functions of this economy.

c. Find a competitive equilibrium allocation and price.

d. Is the competitive equilibrium price unique? Why?

e. Is the competitive equilibrium allocation Pareto efficient? Why?

f. Define a tâtonnement price adjustment process; and define global stability for this economy.

g. Is the equilibrium you found in part c globally stable? Why?