

Department of Agricultural Economics
PhD Qualifier Examination
May 2006

Instructions:

The exam consists of seven questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear in your answer as possible. You have four hours to complete the exam. Be sure to put your assigned letter and no other identifying information on each page of your answer sheets. Also, put the question number and answer page number at the top of each page. Finally, please write on only one side of your paper and leave the appropriate margins.

Good Luck!

(10 points)

1. The following unrelated questions are to be answered with graphs and words.

a. Consider a person who receives utility from food (F) and shelter (S). The person requires a minimum amount of food (F^{\min}) and a minimum amount of shelter (S^{\min}) in order to survive. Let the “just surviving” utility level be denoted as U^S . Show this situation graphically and explain your graph in words. Is it possible for a person to have a higher level of utility and still only consume S^{\min} ? Show this situation graphically and explain your graph in words.

b. Suppose an individual has the utility function $U = q_1 + \alpha q_2$, where $\alpha > 0$. For either good, there are three cases where the substitution and income effects are different. Show these three cases graphically and explain the graph in words.

(15 points)

2. Based on the utility function $U = \beta_1 \ln(q_1 - \gamma_1) + \beta_2 \ln(q_2 - \gamma_2)$ answer the following questions. **Show all work.**

a. Assume $q_i > \gamma_i > 0$, what are the conditions necessary for the preferences to be strictly convex?

b. Derive the Hicksian demand functions.

c. Derive the expenditure function.

d. Derive the indirect utility function.

e. Derive the Marshallian demand functions.

f. Give an economic interpretation of the parameter γ_i in each of the Marshallian demand functions.

g. Prove Slutsky's equation for good one.

(20 points)

3. Consider a competitive industry comprised of $n = 100$ identical firms, each with the average variable cost function, $AVC = q + 10$. In the short-run, labor is the only variable factor; further assume the wage rate $w = 1$. The market demand equation is $D(p) = 1000 - 25p$.

- a. Find the short-run competitive equilibrium.
- b. Calculate total consumer surplus and producer surplus and display graphically.
- c. What is the opportunity cost of producing the equilibrium quantity of output? Show graphically.
- d. How is producer surplus related to profit?
- e. Next, determine the short-run production function, $q = f(L)$; show labor exhibits diminishing returns.
- f. In the long-run, the technology in this industry is characterized by the constant returns to scale production function, $F(L,K)$. Find this function, then show both labor and capital exhibit diminishing returns.
- g. Is this function a member of the CES family? Explain.

(15 points)

4. Amy, Barbara, and Carol are all farmers. They have identical farms and face identical opportunities to make money from the planting of broccoli. Their output depends upon how much it rains, ε . They also must choose the level of fertilizer that they use on their land, z , but fertilizer must be applied before they know how much it rains. Each farm produces broccoli according to the production function $f(z, \varepsilon)$. The price of output is p and the price of fertilizer is w . Each price is taken as exogenous and known. The probability density function for rain is defined by the distribution $g(\varepsilon)$.

Each farmer's utility is a monotonically increasing function of the end-of-the season profits, π . Hence, given choice z_i and rain ε , each of the farmers would earn profits $\pi_i = p \cdot f(z_i, \varepsilon) - w \cdot z_i$ and would enjoy utility, $u_i(\pi_i)$, $i=A, B, C$.

- a. Suppose that Amy maximizes her expected utility. Write down the general form of her optimization problem.
- b. Suppose that Amy is risk neutral and Barbara is risk averse. Under what condition on the production function $f(z, \varepsilon)$ would the fertilizer choice of Amy and Barbara be the same. State the condition at the highest level of generality possible and show why it would be true. (Hint: Consider a specific case for the functional form $f(z, \varepsilon) = \alpha_1 z - .5\alpha_2 z^2 + \beta\varepsilon + \lambda z\varepsilon$. Just considering this case will not yield full credit).
- c. Suppose that Carol has access to a completely accurate weather prediction model and knows how much it will rain. Write down Carol's optimization problem.

(15 points)

5. In 2005, Robert Aumann and Thomas Schelling were awarded the Nobel Memorial Prize in Economics for their work on understanding of conflict and cooperation. Consider a simple situation where a single annual flow of wealth, W , is available for use by two individuals, A and B . The two risk-neutral individuals make their choices simultaneously. If both individuals cooperate, they split the wealth equally. If both parties choose to fight, each party has an equal chance of winning and receiving 100% of the wealth, leaving nothing for the other player. If one player tries to cooperate but the other fights, then a battle ensues and the individual that chose to fight has a probability p_1 that he or she will get 100% of the wealth and a $(1-p_1)$ chance of receiving none of the wealth. If A loses, B receives 100% of the wealth. If there is a battle, both parties pay a cost of c , where $c=0.2 \cdot W$.

- a. If there is only a single-period, under what conditions (i.e., what values for p_1) is fighting an equilibrium outcome? Show your work and explain what you mean by equilibrium.
- b. Now assume that the game is played in two consecutive periods. At the beginning of each period the individuals decide whether to cooperate or fight. Would this repetition change the optimal strategies for the two agents? Explain using the concept of subgame perfect equilibrium. (For simplicity you can assume that their second-period strategies do not depend upon whether they won or lost in the first period.)
- c. One of the reasons that Aumann and Schelling were awarded the Nobel prize was the insight that nations that face each other year after year can achieve a cooperative solution despite the fact that they do not cooperate. Explain this result in the context of the problem considered here. A formal proof is not required.

(15 points)

6. Denote

W : wage rate

Age : age

Exp : working experience

Ed : years of schooling.

Sex : sex ($Sex=1$ if male, and 0 otherwise)

The production function of human capital is sometimes estimated by regressing wage rate on education, age, experience and other relevant variables. For simplicity, suppose the following model is the true model that generates the wage data:

$$W_i = \alpha_0 + \alpha_1 Age_i + \alpha_2 Exp_i + \alpha_3 Ed_i + \alpha_4 Ed_i^2 + \alpha_5 Sex_i + \varepsilon_i,$$

where ε_i is the error term.

- Suppose the model is estimated using the ordinary least squares (OLS). Under what assumptions is the OLS estimate the best linear unbiased estimate (BLUE)?
- Suppose the estimated squared residuals are found to be positively correlated with years of schooling. What assumption of the OLS is violated? What is the consequence of this violation (in terms of estimator properties)? Describe a method to correct this violation.
- What is the marginal effect of education on the wage rate?
- What is the economic interpretation of the coefficient α_5 in the model?
- Suppose the experience variable is not observed for the given dataset. What is the consequence if we estimate the model without using this variable? Since individuals do not begin acquiring experience until they finish their education, a commonly used proxy for experience is

$$Exp^* = Age - Ed - 5.$$

Can we use Exp^* in place of Exp in our regression? Explain your answer.

- Suppose both wage rate and education are positively correlated to an unobservable variable: ability. People with higher ability tend to have more education and higher wage rate. What assumption of the OLS is violated? Describe a method to correct this violation.

(10 point)

7. Consider a coin-toss experiment: the payoff is \$2 if there is a head; and \$1 otherwise. Suppose an unfair coin with a 40% probability of head is used in this experiment.

- a. Calculate the expected payoff of this experiment.
- b. Calculate the variance of the expected payoff of this experiment.
- c. Suppose the probability of head is estimated by

$$p^* = h/n ,$$

where n is number of coin-tosses and h is the number of heads appeared in the experiment. Show that this estimate is consistent.