

**Department of Agricultural Economics**  
**PhD Qualifier Examination**  
**May 2005**

**Instructions:**

The exam consists of six questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear in your answer as possible. You have four hours to complete the exam. Be sure to put your assigned letter and no other identifying information on each page of your answer sheets. Also, put the question number and answer page number at the top of each page. Finally, please write on only one side of your paper and leave the appropriate margins.

Good Luck!

**(20 points)**

1. Consider an economic agent with preferences represented by the utility function

$$u(x) = (x_1^{-1} + x_2^{-1})^{-1}$$

in answering the following.

- a. Are these preferences homothetic? Strictly convex? Strictly monotonic? Explain carefully.
- b. What is meant by “representability,” and what assumptions on preferences are sufficient for representability?
- c. Find the Marshallian demand and indirect utility function.
- d. Compute the Slutsky derivative,  $S_{11}(p,m)$ ; then, knowing  $S_{11}(p,m)$ , explain how one could obtain the remaining Slutsky derivatives.
- e. Are the goods gross substitutes or complements?
- f. Obtain the Hicksian compensated demands and expenditure functions, and explain how Hicksian and Marshallian demands are related, illustrating explicitly for commodity one.
- g. Are the goods net substitutes or complements?

**(20 points)**

2. Suppose that the long-run total cost curve of a representative firm in a competitive industry where an infinite number of potential firms have access to a common cost function  $c(q)$  is given by the formula:

$$c(q) = 1200q - 60q^2 + q^3.$$

- a. Show that when the industry reaches long-run equilibrium the price will be \$300 per unit, and each firm will produce 30 units, if the industry demand curve is  $P^D = 375 - .025Q$ . Show that there will be 100 active firms in long-run equilibrium.
- b. The government wishes to raise \$45,000 by taxing this industry. Two different kinds of taxes are under consideration:
- i) An excise tax of \$15 per unit;
  - ii) A lump-sum tax of \$450 per firm, regardless of how much the firm produces.

Will either of these taxes generate \$45,000 in revenue for the government? Will the new long-run equilibrium price be the same for both tax schemes? If not, will the price increase be greater under the sales tax or the lump-sum tax? Draw a diagram summarizing your answer.

**(10 points)**

3. For the following questions, assume each question is independent of the other questions. For questions a, b, and c establish whether the statement is true, false, or uncertain and why? You are encouraged to use graphs in your answer where appropriate.
- a. Satisfaction of the first and second order conditions for profit maximization implies the variable input use is at its optimal value to maximize profit.
  - b. For a firm with constant returns to scale and positive output, the optimal output level is infinity.
  - c. Increasing marginal cost implies increasing average cost.
  - d. Given the following multiproduct – multifactor production function,  
 $f(y_1, y_2, x_1, x_2) = 0$  where  $y_i$  represents output  $i$  and  $x_i$  input  $i$ , and perfect competition in all product and factor markets, derive all the first order conditions for profit maximization. Give an economic interpretation of the first order conditions.

**(20 points)**

4. There are two types of firms in a market, type 1 and type 2. The total costs of production for all firms of each type are as follows:  $TC_1 = a \cdot q_1$  and  $TC_2 = b \cdot q_2$ , with  $a > b$ . The total quantity demanded for the good ( $q = q_1 + q_2$ ), is a function of the price,  $p$ ,  $q = c - d \cdot p$ .

- a. Assume that there are an infinite number of firms of each type. (i) Identify the competitive equilibrium  $(p, q_1, q_2)$ , where  $q_1$  and  $q_2$  are the aggregate supplies by the two firm types. (ii) Show whether this price-quantity combination is a Nash equilibrium. (iii) Is the outcome Pareto efficient?
- b. Assume that there is only one firm of each type. (i) Identify the Cournot duopoly equilibrium  $(p, q_1, q_2)$ . (ii) Show whether this price-quantity combination is a Nash equilibrium. (iii) Is the outcome Pareto efficient?
- c. Assume that there is only one firm of each type. (i) Identify the Bertrand duopoly equilibrium  $(p, q_1, q_2)$ . (ii) Show whether this price-quantity combination is a Nash equilibrium. (iii) Is the outcome Pareto efficient?
- d. Assume that there is only one firm of each type. (i) Identify the solution if the two firms cooperate to maximize profits  $(p, q_1, q_2)$ . (ii) Show whether this price-quantity combination is a Nash equilibrium. (iii) Is the outcome Pareto efficient?

**(15 points)**

5. Fred and Patty are considering getting married. They both gain utility from their income,  $m$ , and the happiness from being married,  $H_F$  and  $H_P$  respectively.

Fred's utility function is: 
$$u_F = m_F - \left(\frac{m_F}{2}\right)^2 + H_F$$

Patty's utility function is: 
$$u_P = m_P + \left(\frac{m_P}{2}\right)^2 + H_P,$$

where  $H_F$  and  $H_P$  equal zero if they are not married and take on positive values if they are married. Fred earns \$1 every year. Patty's income varies, equaling \$0 half the years and \$2 in other years.

- What is the certainty-equivalent income for Fred and Patty before they get married?
- Assuming that they both agree to get married and that they agree to share their total household income equally, place a lower limit on the values of their marital happiness,  $H_F$  and  $H_P$ .
- If the household utility function for the married couple is  $U_H = \alpha u_F + \beta u_P$ , where  $\alpha$  and  $\beta$  are positive constants, write the formulas for the expected value and variance for household utility in terms of  $u_F$  and  $u_P$ .
- Write the equation for the covariance between  $u_F$  and  $u_P$ . Be as specific as possible, but you do not have to simplify the expressions.

**(15 points)**

6. Answer the following questions

- a. List the assumptions required for ordinary least squares to yield the best linear unbiased estimator.
- b. State the implications for the parameter estimates and their variance estimates for a violation of each one of the assumptions.
- c. Discuss one method for testing each one of the violations.
- d. Discuss one method for correcting each violation problem.