PhD Qualifier Examination

Department of Agricultural Economics

July 31, 2014

Instructions

This exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam.

If an answer requires complicated mathematical calculations, students will be given full credit if they simply write down the function that could have been typed into a calculator.

Important procedural instructions:

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4.1) at the top of each page.
- Write on only **one side** of your paper and the leave at least 1 inch margins on all sides.
- Make sure your writing is clear and easy to read.
- Turn in your final copy with all pages in order.

GOOD LUCK!

1. (15/100) An expected utility maximizer on the space of non-negative amounts of money lotteries has the following utility index, which is explained by the agent's satiation after a high amount of money M.

$$u(x) \equiv \begin{cases} \sqrt{x} & \text{if } x < M \\ \sqrt{M} & \text{if } x \geq M. \end{cases}$$

   (a) Define what is the certainty equivalent of a lottery F for this agent?

   (b) Is there any relation between the certainty equivalent of a lottery, say F, for this agent, and the expected value of F?

   (c) Is this agent risk averse?

   (d) Let F and G be two lotteries such that this agent prefers F to G. Is it necessarily true that F first order stochastically dominates G?

2. (15/100) Suppose we are interested in estimating the following model:

$$y_i = \exp(x_i'\beta)u_i, i = 1, \ldots, n,$$

   where $u_i > 0$, $E[\log u_i|x_i] = 0$, and $E[(\log u_i)^2|x_i] = \sigma^2 < \infty$.

   (a) Present an estimator for this model. Describe the statistical properties of your proposed estimator. State clearly the assumptions required for your results.

   (b) Suppose y denotes the amount of consumption of potato, and one of the covariates included in x is income. Describe how to calculate the income elasticity based on the given model and estimated results.

   (c) Describe how to test the hypothesis that potato is an inferior good. Describe the steps to implement the test.

   (d) Suppose the income variable used in the regression is measured with a classical measurement error with mean zero. Discuss how this measure error may affect the estimates of income elasticities in part (b) and the hypothesis test in part (c)?

3. (15/100) Consider a firm with production function:

$$f(L, K) \equiv AL^{\alpha}K^{1-\alpha},$$

where L is units of labor and K units of capital.

(a) State Sheppard's lemma.

(b) Prove that Sheppard's lemma is satisfied for the production function above (Sheppard's lemma states an equality between two terms that are related with a production function. Here you can use what you know about optimization with Cobb-Douglass functions).

4. (15/100) Consider the following n-good demand system

$$w_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln(X/P),$$

where w_i is the expenditure share associated with the ith good, p_j is the price for the jth good, X is the total expenditure on the system of good given by

$$X = \sum_{i=1}^{n} p_i q_i,$$

where q_i is the quantity demanded for the ith good. P is a linear price index given by

$$\ln P = \sum_{i=1}^{n} w_i \ln p_i.$$

The adding up restriction requires that

$$\sum_{i=1}^{n} \alpha_i = 1, \sum_{i=1}^{n} \beta_i = 0, \sum_{i=1}^{n} \gamma_{ij} = 0.$$

(a) Explain how to incorporate the adding up restriction in your estimation.

(b) Propose an estimator for the demand system given above. Describe the steps to implement your estimator. Discuss the statistical properties of your proposed estimator. State clearly any assumptions needed for your discussion.

(c) Economic theories also suggest homogeneity and symmetry for the demand system. Homogeneity is satisfied if

$$\sum_{j=1}^{n} \gamma_{ij} = 0$$

and symmetry is satisfied if

$$\gamma_{ij} = \gamma_{ji}.$$

i. Explain how to incorporate these restrictions in your estimation. Describe the necessary steps to implement your estimation.

ii. Instead of imposing the restrictions, one might want to test these restrictions on the data. Explain how to test the hypotheses of homogeneity and symmetry in your estimation. Describe the necessary steps to implement your tests.

5. (20/100) Consider a two-agent two-good exchange economy with no free disposal. Agent 1 has a preference relation on $\mathbb{R}^2_+$ given by this utility function: for all $(x_1, y_1) \in \mathbb{R}^2_+$,

$$u_1(x_1, y_1) = x_1 + 2y_1.$$

Agent 2 has a preference relation on $\mathbb{R}^2_+$ given by this utility function: for all $(x_2, y_2) \in \mathbb{R}^2_+$,

$$u_2(x_2, y_2) = 2x_2y_2 - 4.$$

The agents' endowments are $w_1 = (1, 4)$ and $w_2 = (4, 1)$, respectively.

(a) Construct an Edgeworth box diagram showing (and labeling) the endowment allocation and typical preferences or indifference (and directions of increasing preference) curves for each consumer.

(b) If this economy has Pareto optimal allocations, find them and show them clearly on the indifference curves for each consumer. If there are none, state that.

(c) Is the competitive equilibrium allocation Pareto efficient? Why?

(d) If this economy has core allocations, show them clearly on the diagram. If there are none, state that.

(e) Is the competitive equilibrium allocation strictly fair? Why?

(f) Define a tatonnement price adjustment process; and define global stability for this economy.

(g) Is the equilibrium you found in part (c) globally stable? Why?

6. (20/100) Suppose that your department wishes to build a new building. In order to generate a novel design for the building, it sponsors a contest between two architectural firms ($i = 1, 2$). It offers a prize $\Pi > 0$ that will be given to the firm that comes up with the highest quality design (assume that the quality of the design can be objectively evaluated). The losing firm gets nothing. Let $x_i > 0$ denote the quality of the design submitted by firm i and suppose that its cost producing this quality is $C_i(x_i) = x_i^2$. The firms are risk-neutral and the department's ex-post payoff is $\max\{x_1, x_2\} - \Pi$.

(a) Is there a pure strategy Nash equilibrium of this game? If so, derive one such equilibrium.

(b) Consider now a symmetric mixed strategy Nash equilibrium. Let $F(x)$ denote the equilibrium cdf function. Write down the expected payoff of firm i from choosing quality x_i given that firm j adheres to the equilibrium strategy given by $F(x)$.

(c) Derive the equilibrium $F(x)$ representing the mixed strategy Nash equilibrium of this game. What is the support of $F(x)$.

(d) Given Π, derive the equilibrium expected value of $\max\{x_1, x_2\}$.

(e) What value Π^* should the department set.