PhD Qualifier Examination

Department of Agricultural Economics

July 26, 2013

Instructions

The exam consists of **six** questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answers. You have four hours to complete the exam.

If an answer requires complicated mathematical calculations, students will be given full credit if they simply write down the function that could have been typed into a calculator.

Important procedural instructions:

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.

- Also, put the question number and answer page number (e.g. 4.1) at the top of each page.

- Write on only **one side** of your paper and the leave at least 1 inch margins on all sides.

- Make sure your writing is clear and easy to read.

- Turn in your final copy with all pages in order.

GOOD LUCK!

1. (**15 points**) You wish to estimate the following relationship

$$y = \alpha + x\beta + \varepsilon,$$

where x is a scalar.

(a) Derive a method of moments estimator of α and β (denote these as a and b).

You have the following data

y	x
2	1
24	9
-12	-2
-16	-5
-6	-2
22	7
4	2
14	6

For the next two parts, present explicit formulas for your calculations so that you will get credits even if your numerical answers were incorrect.

(b) Based on your method of moments estimator, find a and b.

(c) Calculate the standard error of b. Since you do not have a calculator, you may round your answer to the nearest tenth (0.1).

2. (**20 points**) Let $\succsim$ be a continuous and monotone preference on $\mathbb{R}_+^L$. Recall that the expenditure function and the Hicksian demand associated with $\succsim$ are defined by: for each price vector $p \gg 0$ and each bundle $x^* \in \mathbb{R}_+^L$,

$$e(p, x^*) \equiv \min_{x \succsim x^*} p \cdot x,$$

and

$$h(p, x^*) \equiv \arg \min_{x \succsim x^*} p \cdot x.$$

(a) Argue that e is homogeneous of degree one in p.

(b) Suppose that h is single valued. What is the relation between e and h?

(c) Now consider the function e defined by the following parametric form:

$$e(p, x^*) \equiv \alpha_0 + \sum_{l=1}^{L} \alpha_l \ln(p_l) + \frac{1}{2} \sum_{l=1}^{L} \sum_{k=1}^{L} \gamma_{lk}^* \ln(p_l) \ln(p_k) + \varphi(x^*)\beta_0 \prod_{l=1}^{L} p_l^{\beta_l}, \qquad (1)$$

where φ is an index function and $\alpha_0, ..., \alpha_L, \; \beta_0, ..., \beta_L, \; \{\gamma_{lk}^* : l = 1, ..., L, \; k = 1, ..., L\}$ are constants.

  i. Provide the necessary restrictions for the expenditure function to be homogenous of degree one.

  ii. Derive the Hicksian demand functions associated with the expenditure function.

(d) Suppose that consumers purchase two goods, i and j. Define the 2×2 matrix M as the matrix of Marshallian price elasticities of demand and the vector W as the 2×1 vector of income elasticities of demand. You know 2 of the 6 elements in M and W:

$$M = \begin{bmatrix} \cdot & \cdot \\ -2.4 & 0.6 \end{bmatrix}, \quad W = \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$$

Further, you know that consumers spent 40% of their total expenditure on good i.

  i. Find the 4 remaining elasticities.

  ii. Are these good substitutes or complements of each other?

  iii. Formally demonstrate whether these demand elasticities are the result of a rational preference ordering.

3. (**15 points**) Let $\gamma \equiv (\gamma_l)_{l=1}^L \in \mathbb{R}_+^L$. Suppose that an agent has a utility function on $\{x \in \mathbb{R}_+^L : x \geq \gamma\}$ given by: for each $x \in \mathbb{R}_+^L$,

$$U(x) = \prod_{l=1}^L (x - \gamma_l)^{\beta_l},$$

where $(\beta_l)_{l=1}^L \in \mathbb{R}_+^L$.

(a) Derive the Marshallian demands associated with this utility function.

(b) Derive the Marshallian income and demand elasticities.

(c) Derive the Slutsky substitution matrix.

(d) There is a very simple economist who can only estimate demand under the assumption that consumer preferences are described by this utility function. He can either estimate the demand for Coca-Cola and Pepsi or estimate the demand for shirts and pants. Which pair of goods do you suggest he study? Why?

4. (**15 points**) Consider the standard AR(1) model:

$$y_t = x_t'\beta + e_t,$$

$$e_t = \rho e_{t-1} + u_t, t = 1, \ldots, T.$$

Define $\dot{y}_t = y_t - \rho y_{t-1}$ and $\dot{x}_t = x_t - \rho x_{t-1}$.

(a) Suppose ρ is known. Show that regressing $\dot{y}_t$ on $\dot{x}_t$ for $t = 2, \ldots, T$ using the OLS yields consistent and efficient estimate of β. State clearly the assumptions required for these results.

(b) Suppose ρ is unknown. Propose an estimator for the same model.

(c) Present the large sample properties of the estimator proposed in Part (b). State clearly necessary assumptions for your results.

(d) Suppose now $|\rho| > 1$. Is the estimator proposed in Part (b) still valid? If not, propose an alternative estimator.

5. (**20 points**) Consider a market with inverse demand given by

$$p = 2 - Q$$

where p is price per unit and Q is aggregate quantity demanded. Suppose that this market is served by two quantity-setting firms. Each firm $i = 1, 2$ has a cost function of the form

$$C_i(q_i) = c_i q_i$$

where c_i is firm i's marginal cost, which is drawn independently from a uniform distribution on $[0, 1]$.

(a) Suppose that the two firms observe each other's cost before simultaneously making their output choices. Find the Nash equilibrium of this game $q_i^*(c_i, c_j)$ and the equilibrium price $p_i^*(c_i, c_j)$.

(b) What is the ex-ante equilibrium expected quantity (prior to the cost draw) by each firm? What is the ex-ante expected equilibrium profit by each firm?

(c) Suppose now that c_i is private information to firm i during the quantity setting game. Given a strategy $q_j(c_j)$ for firm j, derive firm i's best response function $q_i^{**}(E[q_j(c_j)], c_i)$.

(d) Suppose that a symmetric BNE exists and denote the corresponding strategy for each firm by $q_i^{**}(\cdot)$. Find a firm's expected output in the symmetric BNE, $E[q_i^{**}(c_i)]$.

(e) What is the symmetric BNE strategy for a firm $q_i^{**}(c_i)$? What is the equilibrium price $p_i^{**}(c_i, c_j)$?

(f) What is firm i's expected equilibrium profit?

(g) Does private information hurt or benefit the two firms?

6. (**15 points**) Consider an exchange economy with two consumers and two commodities x and y. Suppose $X_i = R_+^2$. Each consumer has an income of 1.

(a) Consumer A's preferences are represented by the utility function

$$u_A(x_A; y_A) = x_A^\alpha \times y_A,$$

where $\alpha > 0$. Draw A's indifference curves. What is A's demand function? Graph his demand for x and y as a function of p. Are A's preferences convex? Monotone? Strictly convex? Strictly monotone? Continuous?

(b) Consumer B's preferences are represented by the utility function

$$u_B(x_B; y_B) = (x_B)^2 + 2(y_B)^2.$$

Draw B's indifference curves. What is B's demand function? Graph her demand function for x and y as a function p. Are B's preferences convex? Monotone? Strictly convex? Strictly monotone? Continuous?

(c) Suppose we have an economy with two consumers, both having preferences like consumer 1's, and endowment equal to $(1, 2)$.

   i. Find all the Walrasian equilibria.

  ii. Find the set of core allocations in the Edgeworth Box.

 iii. Find the set of fair allocations in the Edgeworth Box.