

*Corrected VS
Administered
Version*

Department of Agricultural Economics

PhD Qualifier Examination

July 30, 2012

Instructions:

The exam consists of six questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam.

If an answer requires complicated mathematical calculations, students will be given full credit if they simply write down the function that could have been typed into a calculator.

Important procedural instructions:

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Put the question number and page number for that question at the top of each page.
- Write on only one side of your paper.
- Leave at least 1 inch margins on all sides.
- Turn in your final copy with all pages in order.

Good Luck!

1. (15 points) Consider the Cobb-Douglas preference $U(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$, with $0 < \alpha < 1$.
- Given a budget set, $B(p, w)$, derive the Marshallian demand function associated with this preference.
 - Derive the Hicksian demand function associated with this preference and a bundle of goods $\bar{x} = (\bar{x}_1, \bar{x}_2)$.

Now suppose that there is the possibility of a production error on the assembly line. With probability q , the factory will produce defective units of x_1 that yield half the normal utility to the consumer. Unfortunately, the consumer cannot tell whether they have purchased defective units until they actually use them at home. Further, they cannot return defective units for a refund. Thus, if the consumer purchases x_1 units, with probability $(1-q)$ the consumer will enjoy the full x_1 units and with probability q the consumer will enjoy the utility of $x_1/2$ units.

- Derive the *ex ante* Marshallian demand functions for x_1 and x_2 .
- Provide an intuitive reason why the consumer's choices do or do not change as a result of the uncertainty.

2. (20 points) You believe that the functional relationship between height (measured in inches and denoted by y) and age (measured in months and denoted by x) among elementary school children is:

$$y_i = \lambda_i x_i^\beta$$

where x is a scalar and λ_i is an individual-specific unobserved characteristic.

- Propose an estimation procedure to recover the parameter β . Fully describe why you think your proposed method is reasonable and how you would implement it, e.g. if you propose maximum likelihood, explain why you believe it is a sensible method and present the likelihood function.
- Derive the probability limit of $\hat{\beta}$, your parameter estimate proposed in (a). Under what conditions is $\hat{\beta}$ a consistent estimate of β ?

Your dataset, D_1 , does not include the variable y . Instead, D_1 includes $y_i^* = \nu_i y_i$ where ν_i is a random variable. Hence, using the method proposed and maintaining any assumptions provided in (a), you estimate the following relationship:

$$y_i^* = \lambda_i x_i^\gamma$$

- Denote your estimate of γ as $\hat{\gamma}$. Derive the probability limit of $\hat{\gamma}$. What are the least restrictive conditions required for $\hat{\gamma}$ to be a consistent estimate of β ?

For the rest of this question, suppose another researcher has a different dataset, D_2 , that includes y but does not include x . Instead, D_2 includes $x_i^* = \eta_i x_i$, where η_i is a random variable. Using your proposed method and maintaining any assumptions you provided in (a) and (c), the other researcher estimates the following relationship:

$$y_i = \lambda_i x_i^{\delta}$$

- Denote the proposed estimate of δ as $\hat{\delta}$. Derive the probability limit of $\hat{\delta}$.
- Suppose that $E[x_i \eta_i] = 0$. Is this a sufficient condition for $\hat{\delta}$ to be a consistent estimator of β . If yes, explain why. If not, provide conditions for $\hat{\delta}$ to be a consistent estimator of β .
- Under what conditions ~~are~~ does your proposed estimator of δ provide a lower bound on the true value of β .

3. (20 points) Consider an exchange economy with two agents and three commodities. The individual's preferences and initial allocations of the three goods are defined by

$$U^1 = (x_1^1, x_2^1, x_3^1) = \min\left(\frac{x_1^1}{2}, x_2^1, x_3^1\right); \quad \omega^1 = (1, 1, 1)$$

$$U^2 = (x_1^2, x_2^2, x_3^2) = \frac{1}{2} \ln x_1^2 + \ln x_2^2 + \ln x_3^2 \quad \omega^2 = (1, 1, 1)$$

(note that superscripts refer to the individuals and subscripts to the goods).

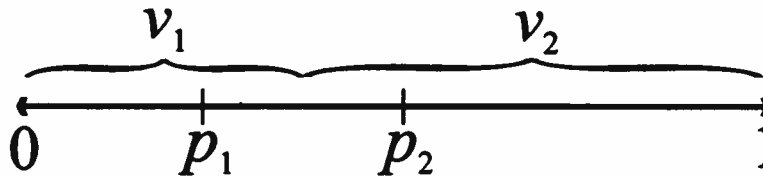
- Find the demand functions for both individuals.
 - Find the aggregate excess demand functions for this economy.
 - Is there an equilibrium in which $p_1=2$ and $p_2=1$?
 - Without identifying the exact value, can it be established that an equilibrium exists?
4. (15 points) Consider the following model

$$Y_{it} = X_{it}'\beta + u_i + e_{it}, \quad i = 1, \dots, n, t = 1, \dots, T,$$

where e_{it} is an iid error with mean zero and finite variance, and u_i is a time invariant individual effect.

- Let $\hat{\beta}_{OLS}$ be the OLS estimator of β . Derive the probability limit of $\hat{\beta}_{OLS}$
 - Under what conditions is $\hat{\beta}_{OLS}$ consistent and efficient?
 - Under what conditions is $\hat{\beta}_{OLS}$ consistent but inefficient?
 - Under what conditions is $\hat{\beta}_{OLS}$ inconsistent?
- Propose an estimator suitable for situation (ii) described in part (a). Describe the steps to implement your proposed estimator.
- Propose an estimator suitable for situation (iii) described in part (a). Describe the steps to implement your proposed estimator.
- Explain why we need to assume that u_i is time invariant in panel data models.

5. (15 points) Two political candidates ($i = 1, 2$) are choosing their campaign positions (p_1 and p_2 , respectively). Their positions are modeled as chosen locations on the real line in the interval $[0, 1]$. One possible set of positions is presented in the figure below. Each candidate's objective is to attract as many votes as possible. Thus, the payoff of candidate i , denoted by v_i , is simply the number of votes that candidate i attracts. There is a continuum of voters, whose favorite positions are uniformly distributed on the interval $[0, 1]$. Suppose that the cost of voting is negligible so that all voters choose to cast a vote. They will vote for the candidate who is closer to their preferred position. If both candidates choose the same position, then they share the votes equally.



- Write down the payoff for candidate i , $v_i(p_i, p_j)$.
- Consider a pure strategy equilibrium (p_i^*, p_j^*) . Can $p_i^* < p_j^*$ be supported as a pure strategy Nash equilibrium? Either characterize such equilibrium or prove that it does not exist.
- Consider a pure strategy equilibrium (p_i^*, p_j^*) . For what values of p_i^* , if any, can $p_i^* = p_j^*$ be supported as a pure strategy Nash equilibrium? If the answer is none, then ~~prove~~ show that this is true.
- Suppose now that the number of candidates has increased to 3. Each candidate is choosing their location. Again, if all candidates choose the same point, they will divide the votes equally. Write down the payoff function for candidate i , $v_i(p_i, p_j, p_k)$.
- Find a pure strategy Nash equilibrium of the three candidates location game or show that such equilibrium does not exist.

6. (15 points) Recall that the equivalent variation from $p \in \mathbb{R}_{++}^2$ to $p' \in \mathbb{R}_{++}^2$ for U at wealth level $w \in \mathbb{R}_+$ is

$$EV(p, p', w) = e(p, x(p', w)) - e(p, x(p, w)),$$

where e is the expenditure function for preference U and x is the indirect utility function.

- a. Show that for p_1, \tilde{p}_1, p_2 , and w (all strictly positive scalars):

$$EV((p_1, p_2), (\tilde{p}_1, p_2), w) = \int_{\tilde{p}_1}^{p_1} h_1(z, p_2, x(\tilde{p}_1, p_2, w)) dz$$

where $h_i(p_1, p_2, x)$ is the Hicksian demand for good i at prices p_1 and p_2 and utility level x . Provide an intuitive explanation why this equality holds.

Now assume that preferences over two goods are Cobb-Douglas:

$$U(q_1, q_2) = q_1^\alpha q_2^{1-\alpha}$$

- b. Derive the indirect utility function, $x(\cdot)$, for these preferences at prices p_1 and p_2 and wealth level w .
- c. Show that the Hicksian demand functions at prices p_1 and p_2 and utility level indirectly defined by $x(\tilde{p}_1, p_2, w)$ satisfy

$$h_1((\tilde{p}_1, p_2), w) = \alpha \frac{w}{\tilde{p}_1^\alpha p_1^{(1-\alpha)}} \text{ and } h_2((\tilde{p}_1, p_2), w) = (1-\alpha) \frac{w p_1^\alpha}{\tilde{p}_1^\alpha p_2}$$

- d. Verify that (a) holds for Cobb-Douglas preferences. (Here you have to calculate EV from its definition and verify that it is equal to the right side of the equation in (a).)