

Department of Agricultural Economics

PhD Qualifier Examination

August 2011

Instructions:

The exam consists of six questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear as possible in your answer. You have four hours to complete the exam.

If an answer requires complicated mathematical calculations, students will be given full credit if they simply write down the function that could have been typed into a calculator.

Important procedural instructions:

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4.1) at the top of each page.
- Write on only **one side** of your paper and leave at least 1" margins on all sides.
- Turn in your final copy with all pages in order

Good Luck!

In answering some parts of the next question, it may be helpful to recall that:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

1. (20 points) An applied econometrician wishes to estimate the following relationship:

(i)
$$y_i = x_i \beta + z_i \gamma + \varepsilon_i$$

where x_i and z_i are scalars (assume the data have been demeaned to remove the constant).

- a. Define the matrix $X = [x \ z]$, where x and z are $n \times 1$ vectors (X is an $n \times 2$ matrix). Express in the simplest possible way each element of the matrices

$$\text{plim} \left[\left(\frac{1}{n} \right) X' X \right] \text{ and } \text{plim} \left[\left(\frac{1}{n} \right) X' \varepsilon \right]$$

as n goes to infinity.

- b. Suppose that the econometrician estimates equation (i) using OLS. Derive the probability limit of β (no credit will be given for leaving the probability limit in matrix notation). Call this β_1 .
- c. Suppose $E[x\varepsilon]=0$ and $E[z\varepsilon] \neq 0$. Provide conditions under which β_1 is a lower bound on the true value of β .
- d. Suppose that instead of estimating (i), the econometrician estimates the following using OLS:

(ii)
$$y_i = x_i \beta + v_i$$

where $v_i = z_i \gamma + \varepsilon_i$. Derive the probability limit of β (no credit will be given for leaving the probability limit in matrix notation). Call this β_2 .

- e. Provide conditions under which β_2 is an upper bound on the true value of β when β_1 is a lower bound on the true value of β .
- f. Provide a real-world example (i.e. provide examples of y , x , and z) where estimating (i) and (ii) using OLS would yield upper and lower bounds on the true value of β .

2. (15 points) Consider the following parametric form for a candidate cost function for a firm:

$$C(y, w) = \min \left\{ d \left[y \left(\frac{bw_1 + w_2}{a} \right)^a \left(\frac{w_3}{c} \right)^c \right]^{\frac{1}{d}}, hw_4 y^g \right\}$$

where y is the firm's output, which is produced using inputs x_1, x_2, x_3 , and x_4 , with prices w_1, w_2, w_3 , and w_4 .

- Find restrictions on the scalars a, b, c, d, g and h such that $C(\cdot)$ is a valid cost function for some convex monotonic technology.
- Find a form of production function, $F(x_1, x_2, x_3, x_4)$ that generates $C(y, w)$ as its cost function.

3. (15 points) Consider the following system of equations:

$$Y_1 = X_1' \beta_1 + u_1,$$

$$Y_2 = X_2' \beta_2 + u_2$$

where Y_j is a $n \times 1$ vector, X_j is a $n \times k_j$ matrix, u_j is a $n \times 1$ vector for $j = 1, 2$. It is assumed that $E[u_j] = 0$, $Var(u_j) = \sigma_j^2 I_n$, $0 < \sigma_j^2 < \infty$ for $j=1,2$, where I_n is a n -dimensional dimensional identity matrix, and $Cov(u_1, u_2) = \sigma_{1,2} I_n$, σ_1^2 , σ_2^2 and $\sigma_{1,2}$ are unknown parameters.

- Construct a simultaneous equation estimator of the two equations. Present the estimator explicitly in a matrix form.
- Present a feasible efficient simultaneous equation estimator.
- Suppose that $\sigma_{1,2} = 0$. Discuss the relation between the efficient simultaneous equation estimator given in part (b) and the estimators obtained by simple OLS on the two equations separately.
- Suppose that $\sigma_{1,2} \neq 0$. Discuss the relation between the efficient simultaneous equation estimator given in part b and the estimators obtained by simple OLS on the two equations separately.

4. (15 points) Consider a public goods economy with one private good, x , one public good, y , and n agents. Each agent's consumption set is the nonnegative quadrant. Agent i is endowed with w_i units of private good, and his preference is denoted by \succsim_i . There are no initial endowments for the public good, but the public good can be produced by using private contributions to the public good according to a production technology $y = v/q$ with $q > 0$, $v = \sum_{i=1}^n g_i$ and $x_i + g_i = w_i$.
- Define the Lindahl equilibrium AND Pareto efficiency for this economy.
 - Prove every Lindahl equilibrium allocation is Pareto efficient. If any additional assumption is needed, state it explicitly and show where it is used in the proof.
 - Now suppose preferences \succsim_i of agent i can be represented by utility function $u_i(x_i, y)$ that are differentiable and have positive marginal utilities. Derive the first order condition(s) that are necessary for interior Pareto Optimality. (Interior here means that x_i and y are strictly positive numbers.) Note: Lagrange multipliers should not appear in the final form of the condition(s).
 - When utility functions of agents are given by $u_i(x_i, y) = x_i^{\alpha_i} y^{\beta_i}$ with $\alpha_i > 0$ and $\beta_i > 0$, find the Lindahl equilibrium. Is it Pareto optimal?
5. (15 points) An expected utility maximizer over lotteries of money has utility index u and initial wealth, w . Lottery 1 offers a payoff of $x > 0$ dollars with probability α and a payoff of $y > x$ dollars with probability $1 - \alpha$.
- If the agent owns the lottery, what is the minimum price \bar{p} at which she is willing to sell it (find the equation that characterizes \bar{p}).
 - If the agent does not own the lottery, what is the maximum price \underline{p} that she is willing to pay for it (find the equation that characterizes \underline{p}).
 - If the agent is risk averse and exhibits non-increasing relative risk aversion, what can we say about the relation between \underline{p} and \bar{p} ?
 - If the agent is risk averse and exhibits non-increasing absolute risk aversion, what can we say about the relation between \underline{p} and \bar{p} ?

6. (20 points) A firm is considering hiring a worker, who can be one of two types: high or low (i.e. $\theta = \{H, L\}$). The worker knows his own type and the firm only knows that the probability of the worker being of high type is $\frac{1}{3}$. The high ability worker generates a revenue of $\pi(H) = 2$ for the firm and the low ability worker- $\pi(L) = 0$. If hired, the firm will pay the worker a fixed wage $w = 1$. The high ability worker has the possibility of an alternative occupation earning him a payoff of $\frac{1}{2}$ while the value of the low ability worker's outside option is 0.
- a. Let $h = \{1, 0\}$ denote the firm's hiring decision where 1 stands for the firm's choice to make a job offer to the employee. Specify h^* in a Bayesian Nash Equilibrium (BNE) of this game.

Suppose now that before the firm makes its hiring decision, the worker can choose to invest in education. Let $e = \{1, 0\}$ denote the education strategy of the worker where $e = 1$ stands for the worker's choice to acquire education. The cost for the high type of acquiring education is $c(1|H) = \frac{1}{6}$ and the cost for the low type of acquiring education is $c(1|L) = \frac{2}{3}$. While the education does not affect the workers' productivity inside or outside the firm, it is observable to the firm. Let $\mu(\theta|e)$ denote the firm's belief that the worker is of type θ after observing the education choice.

- b. Draw a game tree depicting the extensive form of this game.
- c. Specify a fully separating Perfect Bayesian Equilibrium (PBE) or explain why such equilibrium does not exist.
- d. Specify a pooling PBE or explain why such equilibrium does not exist.
- e. Specify a (non-degenerate) partially separating PBE $(\alpha(e|\theta), \hat{h}(e), \mu(\theta|e))$, where $\alpha(e|\theta)$ stands for the probability of type θ choosing e and $\hat{h}(e)$ stands for the probability of the firm making a job offer to the worker after observing e , or explain why such equilibrium does not exist.