Department of Agricultural Economics

PhD Qualifier Examination

August 2010

<u>Instructions:</u>

The exam consists of six questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear in your answer as possible. You have four hours to complete the exam.

If an answer requires complicated mathematical calculations, students will be given full credit if they simply write down the function that could have been typed into a calculator.

Important procedural instructions:

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4.1) at the top of each page.
- Write on only **one side** of your paper and the leave at least 1” margins on all sides.
- Turn in your final copy with all pages in order

Good Luck!

1. (10 points) Consider the infinitely repeated Bertrand (price competition) oligopoly, that is, a homogeneous product market with downward-sloping demand and constant (identical) marginal costs of production c for each of the n firms. Suppose that the monopoly price p^m is unique. Let r denote the discount rate for the infinitely repeated game and denote the monopoly profit by π^m.

   a. Characterize the Nash equilibrium strategies for each player in the stage game.

   b. Show that when $r \leq \frac{1}{n}$ any price in the interval $[c, p^m]$ can be supported as a (stationary) subgame perfect equilibrium, while when $r > \frac{1}{n}$ the Nash equilibrium strategy of the stage game is played in every stage of any subgame prefect equilibrium path.

2. (15 points) Consider a pure-exchange economy with two consumers, A and B and with two commodities, x_1 and x_2.

$$u_A = \min\left[x_A^1, x_A^2\right], \omega_A = (3,0)$$
$$u_B = \min\left[x_B^1, x_B^2\right], \omega_B = (0,3)$$

   Both consumption sets x_A and x_B consists of integer points in $\mathbb{R}_+^2$; i.e. both commodities can only be consumed in whole units.

   a. Provide two examples of real-world goods that could exhibit the same consumption set restriction as that imposed on x_A and x_B

   b. Show that the price vector $p = (p_1, p_2)$ with $p_1 = p_2$ is a competitive equilibrium price. Calculate all competitive equilibrium allocations associated with this price vector.

   c. Show whether (each of) the competitive equilibria is a (regular) Pareto optimum.

   d. State the First Theorem of Welfare Economics. Discuss how results of part (c) conform with or violate the First Theorem.

3. (20 points) Suppose you are hired to estimate the following relationship:

$$y_i = a + bx_i + u_i$$

where the continuous variable *x*, the coefficients *a* and *b*, and the unobservable *u* are all scalars. No assumptions are made about the relationship between *x* and *u*.

a. What are the necessary assumptions on data generating processes of *y*, *x* and *u* for the probability limit of *b* from OLS estimation to be well-defined? Under these assumptions, what is the probability limit of the OLS estimate of *b*?

b. Suppose *y* is farm yield (bushels of corn per acre) and *x* is the health status of the farmer (*x*=1 if the farmer is perfectly healthy, *x*=0 if the farmer is perfectly sick). Do you expect *b*, the true value, to be positive, negative or zero? Provide an explanation for why you believe this to be the case.

c. What would have to be true for the OLS estimate of *b* to be a consistent estimate of *b*. Do you expect the OLS estimate of *b* to equal (asymptotically) the true value, to be larger than the true value or to be smaller than the true value? Provide an intuitive explanation for why you think so.

d. Unfortunately, you do not have access to STATA or GAUSS. Instead, you only have a version of Excel that allows you calculate the mean, variance and covariance between different variables (of course, you can use all the standard operators like adding and subtracting variables).

   How could you estimate the coefficients *a* and *b* using OLS when *x* is continuous? [Hint: you definitely want to estimate *b* first].

e. Suppose you are concerned about the consistency of your OLS estimate and decide to use IV regression with scalar *z* as the instrument for *x*. Show that the 2SLS estimate of *b* is b_y/b_x where:

   $x_i = a_x + b_x z_i + \varepsilon_i$

   $y_i = a_y + b_y z_i + \mu_i$

   [Hint: the 2SLS estimate of *b* in matrix notation is $(Z`X)^{-1}Z`Y$.]

4. (15 points) Let $X \equiv \mathbb{R}^3$ be the set of alternatives. Suppose that a demand function x satisfies Walras' law and is such that for each pair $(p, w) \in \mathbb{R}^3_{++} \times \mathbb{R}_+$

$$x_1(p,w) \equiv 100 + \frac{1}{10}\frac{p_1}{p_3} + 20\frac{p_2}{p_3},$$

$$x_2(p,w) \equiv 450 + 20\frac{p_1}{p_3} - \frac{5}{3}\frac{p_2}{p_3}.$$

   a. Derive an expression for $x_3(p,w)$. You do not need to simplify the expression.

   b. Is the demand vector x homogeneous of degree zero?

   c. Write the general form for the lk^{th} component of the Slutsky matrix associated with the demand vector x, $S_{lk}(p,w)$, in terms of the partial derivatives of x. Calculate $S_{11}(p,w)$, $S_{12}(p,w)$, $S_{21}(p,w)$, and $S_{22}(p,w)$.

   d. Is x the result of preference maximization (for some monotone preferences)?

5. (20 points) Consider a random sample of 3 independent observations from a Poisson distribution: X_1=3, X_2=0, X_3=1. The density for each observation is

$$f(x,\theta) = \frac{e^{-\theta}\theta^x}{x!}$$

   a. Write down the likelihood function of a maximum likelihood estimator for the given sample. Calculate the MLE estimate of θ.

   b. Calculate the standard error of the MLE estimate of θ..

   c. Design a test for the null hypothesis $H_o : \theta = 1{:}5$.

   d. It is known that a Poisson random variable with density function given above has both mean and variance being θ. Design a generalized method of moment estimator for θ based on the sample mean and variance.

6. (20 points) Consider a market with risk averse agents, each with utility function $U(w)=\ln(w)$, and a risk neutral insurer, where w is end of period wealth. Suppose that with probability 0.5 the agent's end of period wealth is w_1, and with probability 0.5 the end of period wealth will be $w_1 - Z$, where Z is the cost incurred by the agent due to a particular observable loss event.

   Suppose the insurer offers an insurance contract which will pay the agent if the observable loss event occurs and zero if the loss event does not occur. The unit price of the insurance contract is P. The agent can choose a quantity A=[0,1] units of the insurance contract to buy. Thus, the cost of the insurance is $P \times A$, and the amount the insurer pays (i.e., indemnifies) the agent in the event of loss is $Z \times A$.

   a. Set up the agent's utility maximization problem.

   b. Derive the first-order condition and find an equation for the optimal quantity of insurance the utility maximizing agent would buy. Give an interpretation of what this is and what it means.

   c. We know that when insurance is priced actuarially fair (meaning that the expected payout/indemnity of the insurance contract equals the premium charged) that risk averse agents will fully insure. Express the actuarially fair premium in terms of Z, P, and other information above. Using the solution to (b) above, show that when premiums are actuarially fair, risk averse utility maximizing agents fully insure.

   d. Suppose the insurer is a profit maximizing risk neutral firm (for simplicity, assume the insurer's only costs are from paying claims for Z). Set up the insurer's profit maximization problem assuming that the insurer can choose the price of insurance, P.

   e. Find the first order conditions for the insurer to maximize profit. Solve. Explain.