Department of Agricultural Economics

PhD Qualifier Examination

August 2009

Instructions:

The exam consists of six questions. You must answer all questions. If you need an assumption to complete a question, state the assumption clearly and proceed. Be as clear in your answer as possible. You have four hours to complete the exam.

If an answer requires complicated mathematical calculations, students will be given full credit if they simply write down the function that could have been typed into a calculator.

Important procedural instructions:

- Be sure to put your assigned letter and no other identifying information on each page of your answer sheets.
- Also, put the question number and answer page number (e.g. 4.1) at the top of each page.
- Write on only **one side** of your paper and the leave at least 1” margins on all sides.
- Turn in your final copy with all pages in order

Good Luck!

1. (20 points) Consider an economy with two-persons, i=1,2, each of whom consume two commodities, x_{i1} and x_{i2}. The feasible consumption set is restricted as follows:

$$X_i = \{x_i \in R^2_+; x_{i1} + x_{i2} \geq 1\}, \qquad i = 1,2$$

The initial endowments for the two individuals are: $w_1 = (1, 0)$ and $w_2 = (1, 2)$.

The utility for the individuals are:

$$U_1(x_{11}, x_{12}) = x_{11} + 2x_{12}, \text{ and}$$
$$U_2(x_{21}, x_{22}) = \ln(x_{21}) + \ln(x_{22})$$

a. Draw the Edgeworth Box with the consumption set, indifference curves, and the initial endowment point clearly marked.

b. Calculate demand functions for agents 1 and 2.

c. Find the set of Pareto optimal allocation and draw them in the Edgeworth Box.

d. Does a competitive equilibrium exist? If so, is it a Pareto optimal? If not, which of the standard assumptions for the existence theorem is violated?

e. Change the consumption set to $X_i = \{x_i \in R^+_+\}$, $i = 1,2$.

   Does the change make a difference in your answer to part (d) above?

2. (20 points) Consider a one shot price competition (Bertrand) model with profit maximizing firms, i, that set prices, p_i and can meet all resulting demand. Let $N = \{1,2, \dots , n\}$ denote the set of firms. Let k indicate the number of firms that set the lowest price $\overline{p}$ and let $D(\overline{p})$ be a downward sloping demand curve. These firms split the resulting demand equally and the remaining firms sell zero units. That is, if $k = \#\{j \in N \mid p_j = \overline{p}\}$, then the sales of firm i are $q_i = \frac{D(\overline{p})}{k}$ if $p_i = \overline{p}$,and $q_i = 0$ otherwise.

   a. Suppose fixed costs are zero and marginal costs equal c for all firms. Characterize the Nash Equilibria (price, quantity, and k).

   b. Compare the properties of the equilibria found in part a with the cases of Monopoly and Perfect Competition.

   c. Suppose a Monopoly would be strictly viable and that there is a fixed production cost F that can be avoided if production shuts down entirely.
   Let the cost function be $C(q_i) = F + cq_i$ if $q_i > 0$ and $C(0) = 0$. Characterize conditions under which an equilibrium does or doesn't exist.

   d. Does your answer to part c change if you allow mixed strategies? If so, how?

3. (15 points) Suppose that a correctly specified regression would be

$$y = X_1\beta_1 + X_2\beta_2 + \varepsilon,$$

where the two parts of X have K_1 and K_2 columns, respectively, $E\left[\varepsilon \mid X\right] = 0$ and $Var\left(\varepsilon \mid X\right) = \sigma^2$.

a. Suppose that we regress y on X_1, without including X_2. Denote the estimate $\tilde{b}_1 = \left(X_1'X_1\right)^{-1} X_1'y$.

   Derive the bias of $\tilde{b}_1$.

b. Derive $Var\left[\tilde{b}_1 \mid X_1\right]$.

c. Suppose that we now compute the correct model, including X_2. Let the estimate for β_1 be $\hat{b}_1$, which is unbiased and has a variance

$$Var\left[\hat{b}_1 \mid X\right] = \sigma^2\left[X_1'X_1 - X_1'X_2(X_2'X_2)^{-1}X_2'X_1\right]^{-1}.$$

   Compare the variances of $\hat{b}_1$ and $\tilde{b}_1$. Show that the biased estimate, $\tilde{b}_1$, may nevertheless exhibit a lower mean-squared error.

4. (10 points) Consider the conditional distribution:

$$f(y \mid x) = \frac{1}{\alpha + \beta x} e^{-y/(\alpha+\beta x)}, \ \ y \geq 0, \ \ 0 \leq x \leq 1.$$

a. Derive the conditional mean $E[y \mid x]$. (Hint: If w is an exponential random variable with density function $\lambda e^{-\lambda w}$., then $E(w) = 1/\lambda$ and $Var(w) = 1/\lambda^2$.)

b. Suppose that x is uniformly distributed between 0 and 1. Derive $E(y \mid x)$ and $Var(y \mid x)$.

c. Derive $Cov[x, y]$. (Hint: You can evaluate the covariance directly. Alternatively, you can use the fact that $Cov[x, y] = Cov[x, E[y \mid x]]$.)

5. (20 points) Suppose a firm has production function with two inputs, z_1 and z_2:

$$f\left(z_1, z_2\right) = k + \alpha \ln\left(z_1\right) - \frac{\beta}{\left(z_2\right)^2}$$

where $k > 0$, $\beta > 0, \alpha > 0$. Assume that the input prices are constant at w_1 and w_2.

a. Is it possible that only input 1 or 2 will be used in production? Explain.

b. Suppose for some α, β, and z_1' and z_2', we have $f\left(2z_1', 2z_2'\right) = 3f\left(z_1', z_2'\right)$. Is this an example of constant, decreasing or increasing returns to scale?

c. Provide restrictions on α, β, and k that guarantee the production function exhibits decreasing returns to scale as z_1 and z_2 go to infinity.

d. Set up and solve the profit maximization problem, deriving the factor demand functions.

e. Derive the optimal profit and optimal output level functions.

f. Set up the cost minimization problem and provide the first-order conditions.

g. The production function does not yield a closed form solution. Therefore, use the first-order conditions to implicitly define the conditional factor demand for z_2 as a function of α, β, k, w_1, w_2 and the desired output level, q.

h. Demonstrate that the conditional factor demand for z_2 is increasing in the output level, increasing in w_1 and decreasing in w_2.

6. (15 points) Consider the following utility functions defined over *levels of wealth*, x. The function

$$u(x) = -\alpha e^{-\alpha x} + \beta \tag{1}$$

exhibits constant absolute risk aversion (CARA), while the function

$$u(x) = \frac{\alpha x^{1-\rho}}{1-\rho} \tag{2}$$

exhibits constant relative risk aversion (CRRA) with $\alpha > 0$.

a. For what values of α is the CARA function risk averse?

b. For what values of α and ρ is the CRRA function risk averse?

c. Write an expression for the certainty equivalent for a 50/50 gamble between $0 and $100 for each utility function.

d. Consider an individual with CARA utility (1) with initial wealth, w. She is facing a lottery in which there is a 50% chance of being paid $100 and 50% of receiving $0. Show that the certainty equivalent for this lottery is the same for any level of $w > 0$.

e. Consider an individual with CRRA utility (2) with initial wealth, w. She is facing a lottery in which there is a 50% chance receiving a payment equal to 10% of w and a 50% chance of receiving $0. Show that the ratio of the certainty equivalent to w (CEq/w) is the same for any level of $w > 0$.