Department of Agricultural Economics
PhD Qualifier Examination
July 31, 2007

The exam consists of six questions. You must answer all questions. You have four hours to complete the exam.

If you need an assumption to complete an answer, state the assumption clearly and proceed with your answer. Label each answer page you produce with your assigned letter identification. In addition place the question number on each sheet (page) of paper. Do not put any other identifying information on the answer pages. Write on just one side of the page and leave sufficient margins for copying and grading comments.

1. (15 points) Answer all parts:

(i) Evaluate:

(a) For an economic agent for which all goods are gross substitutes, the demand for every good will be price elastic.

(b) Quasi-linear preferences will result in exactly one good being a luxury good.

(ii) A consumer facing price vector $p^1 = (5,10)$ for two goods selects quantities $x^1 = (20,40)$, where order corresponds to good one and good two, respectively. Under a new set of prices $p^2 = (l0,5)$ his consumption vector is $x^2 = (\theta,30)$. For what range of values for θ will the weak axiom of revealed preference be violated? What can be said about all other values of θ?

(iii) From the identity, $E(p,v(p,m)) \equiv m$, generate the relation between Marshallian and Hicksian demands, then derive the Slutsky decomposition of the price effect into income and substitution effects.

2. (20 points) Given the normalized profit function, $\pi(w) = w_1^{-½} + w_2^{-½}$, find the following:

a. the supply function.
b. derived demands.
c. the cost function.
d. the conditional input demands.
e. the production function.
f. the elasticity of substitution from the production function, then confirm this result from the cost function.
g. Finally, explain how returns to scale can be determined from either the production function or the cost function.

3. Let $X_1, X_2, \ldots X_n$ be an i.i.d. random sample from a distribution which takes value 0 with probability p, and takes value one with probability $1-p$, where $O<p<1$.

    a. Show that Prob.$(X_1=1)$=E(X_1). [5 points]

    b. Derive the variance of X_1. [5 points]

    c. Denote $Z=(X_1 + X_2+...+ X_n)/n$. Derive the expectation and variance of Z. [5 points]

4. Consider the random variable $X_1, X_2, \ldots, X_n$ which only takes the values of zero or one. Suppose we are to predict X using another variable Z.

   a. Consider the simple linear model: $X_i = \alpha + \beta Z_i + e_i$, where e_i is an error term with zero mean and finite variance. Discuss the limitations of the ordinary least squares estimator of β. [5 points]

   b. Suppose we use, instead, the model $X_i = \dfrac{\exp(a+bZ_i)}{1+\exp(a+bZ_i)}$. What is the range of the predicted value? Explain how to calculate the marginal effect of Z on X. [5 points]

   c. The log odds ratio is defined as q=log(p/1-p), where p is the probability of X=1. Derive the marginal effect of Z on the log odds ratio q. [5 points]

5. (15 points) Two farmers Alice (A) and Betty (B) are deciding how much of their land to plant. Alice and Betty each have 1000 acres of land and they can plant corn and/or tomatoes. Corn is very reliable; it yields $20 per acre every year. Tomatoes are more risky but more lucrative; an acre of tomatoes yields $60/acre in good years, which happen 40% of the time and $10/acre in bad years which happen 60% of the time.

The farmers get ex-post utility as a function of their wealth at the end of the season as follows:

Alice: $U_A = -10 + w_A$

Betty: $U_B = 10 \times \ln(w_B)$

where w_A and w_B are the farmers' income at the end of the year, measured in thousand of dollars.

Each farmer must decide what crops to plant. She can allocate part of her crops to Corn and part to Tomatoes, or all to one or the other. However, their choices must be made before they find out what kind of year it is going to be. Let α_A and α_B be the portion of each farmer's land planted in Corn.

(a) Write out the expected utility problem that each farmer would solve.
(b) Identify the portion of crops that each farmer would put in Corn.
(c) Provide an economic explanation as to why the two farmers would make different choices.
(d) Suppose a company is interested in renting land in the area. The company is willing to pay $22.50 to rent the farmers' land. Show whether Alice and/or Betty would accept the offer.

6. (20 points) Two players must decide whether to play hide-and-seek game. Ifboth are hiders, both receive zero; if both are seekers both receive I (at least they see each other). If one is seeker and the other one is hider, the seeker receives 3 and the hider 2.

(a) Find the normal form of this game.
(b) Find the Nash equilibria of this game.
(c) Are there any dominated strategies?
(d) Find the pure and mixed strategy Stackelberg equilibria in which player 1 moves first.

Now suppose that the game is infinitely repeated. Player 1 is a long-run player with discount factor $\beta=1/(1+r)$. Player 2 is a short-run player with discount factor zero ($r = \infty$).

(e) Find the range of payoffs in each repetition for the long-run player that can be supported in a sub-game perfect equilibrium.
(f) Find strategies for the long- and short-run players that support the best equilibrium from Part e.