

Department of Agricultural Economics
PhD Qualifier Examination
January 2007

Instructions:

The exam consists of six questions. You must answer all questions. You have four hours to complete the exam.

If you need an assumption to complete an answer, state the assumption clearly and proceed with your answer. Label each answer page you produce with your assigned letter identification. Do not put any other identifying information on the answer pages. In addition place the question number on each sheet (page) of paper. Write on just one side of the page and leave sufficient margins for copying and grading comments.

1. Consider the quasi-linear utility function, $u(x) = x_1 + 2\sqrt{x_2}$: (20 points)

(a) Find the Marshallian demands, using care to partition the domains to distinguish corner solutions from interior solutions.

(b) When **both** goods are purchased, answer the following. Apply the definitions of gross substitutes and complements to these two goods. Are they normal or inferior? Is either a luxury good? Compute the Slutsky derivatives, $S_{11}(p,m)$ and $S_{22}(p,m)$, and show $p_1^2 S_{11} = p_2^2 S_{22}$, where $p = (p_1, p_2)$ are prices and m income available.

(c) Are these goods net substitutes or complements? Are the underlying preferences (strictly) convex? Explain carefully.

(d) Sketch graphs of both Engel curves; again, be careful with the partitioning of domains and label your graphs appropriately.

2. Consider a competitive industry comprised of $n = 50$ identical firms, each with the average variable cost function, $AVC = q + 8$. In the short-run, labor is the only variable factor; further assume the wage rate $w = 1$. Market demand is $D(p) = 600 - 15p$. **(a)** Find the short-run competitive equilibrium. **(b)** Calculate total consumer surplus and producer surplus and display graphically. **(c)** What is the opportunity cost of producing the equilibrium quantity of output? Show graphically. How is producer surplus related to profit? **(d)** Suppose this market is serviced by a monopolist with the same cost structure, $AVC = q + 8$. Find the profit maximizing levels of price (p_m) and quantity (q_m). Also determine the consumer and producer surplus for this monopoly case. **(e)** Compare the sum of consumer and producer when this is a competitive industry with the monopoly scenario. Why is this result to be expected? **(f)** Next, determine the short-run production function, $q = f(L)$; show labor (L) exhibits diminishing returns. **(g)** In the long-run, the technology in this industry is characterized by the constant returns to scale production function, $F(L,K)$, where K is capital. Find this function. (20 points)

3. An individual's utility as a function of ex post wealth is $u = -e^{-\gamma W}$. The individual faces a loss of $\$L$ if it rains, which occurs with probability p . That is, $W = W_0 - L$ with probability p and $W = W_0$ with probability $1 - p$. She has the opportunity to purchase an insurance policy that pays $\$X$ if it rains. The price of the insurance policy is θpX . What is the optimal amount of insurance that the individual would purchase? (15 points)

4. Consider a random sample $\{y_t, x_t\}$, $t=1, 2, \dots, T$, where y_t is a scalar and x_t is a K by 1 vector. Suppose the data generating process of y_t is the following:

$$y_t = \alpha + x_t \beta + u_t,$$

$$u_t = \rho u_{t-1} + \varepsilon_t,$$

where $|\rho| < 1$ and ε_t is an i.i.d. error term with mean zero and finite variance σ^2 (20 points):

- (a) It is known that ordinary least squares (OLS) estimator is the Best Linear Unbiased Estimator (BLUE) under some regularity conditions. If you estimate the above model using OLS, what properties does your estimator have? Explain your answer, especially discuss what regularity condition(s) are violated?
- (b) Suppose the value of ρ is known. Describe a method to deal with the problem in part (a). Discuss the properties of the estimator you propose.
- (c) Suppose the value of ρ is unknown. Describe a method to implement the method you suggest in part (b). Discuss the properties of the estimator you propose.
- (d) What is the consequence if we remove the assumption $|\rho| < 1$?

5. You are working as a price analyst for a feeder cattle operation in west Texas. You are positioned at time t and seek a forecast of price for period $t+1$. You have three types of unbiased forecasts (made by different models or people) of feeder cattle prices available to you: $f_{1,t+1}$, $f_{2,t+1}$, and $f_{3,t+1}$. Each forecast type has a historical forecast error variance given respectively as: σ_1^2 , σ_2^2 , and σ_3^2 . Corresponding covariances are given as $\sigma_{1,2}$, $\sigma_{1,3}$, and $\sigma_{2,3}$.

Derive the weights λ_1 , λ_2 , and λ_3 that can be used to find an unbiased linear combination of the forecasts that minimizes the overall variance of the forecast error for period $t+1$. If you use matrix methods you can leave your answer in matrix inverse form, you do not have to solve for the inverse. (10 Points).

6. Consider a market with two identical firms. Marginal cost is constant, given by m . Demand is given by $P(X) = \alpha_0 - \alpha_1 X$, where $X = X_1 + X_2$, is the sum of the two firm's output quantity. (15 points)

- a. If the two firms collude, choosing output jointly, what would be the aggregate output and market clearing price?
- b. If the two firms both act as if they are in a purely competitive market, what would be the aggregate output and market clearing price?
- c. What would be the Cournot equilibrium aggregate output and market clearing price?
- d. What would be the Bertrand equilibrium aggregate output and market clearing price?
- e. Of the four solutions identified above, which are Nash equilibria? Explain.
- f. Each firm's profit, fixed cost are F_i , $i=1,2$.